

1.4 Beginning Proof

Objective

After studying this section, you will be able to

- Write simple two-column proofs

Geometric proofs can be written in different ways. In this class, you will learn two column and paragraph formats. A paragraph proof is only a two-column proof written in sentences. However, since it is easier to leave steps out when writing a paragraph proof, we'll learn the two-column method.

A two-column geometric proof consists of a list of statements, and the reasons that we know those statements are true. The statements are listed in a column on the left, and the reasons for which the statements can be made are listed in the right column. Every step of the proof (that is, every conclusion that is made) is a row in the two-column proof. Each statement can only have one reason; if you need more than one reason, you need more than one statement.

Chain of reasoning is essential in writing proof.

Writing a proof consists of a few different steps.

1. Draw the figure that illustrates what is to be proved. The figure may already be drawn for you, or you may have to draw it yourself.
2. If you have a lot of givens, MAP it. Your proof should be well organized and easy to follow. Remember that you write for someone else to follow your argument. List a given statement, and infer what you may from it. The STATEMENT column "verifies" the truth of the statement, and the REASON explains "why" it is true. Only one definition, postulate, or theorem may be used per line.
3. Mark the figure according to what you can deduce about it from the information given. This is the step of the proof in which you actually find out how the proof is to be made, and whether or not you are able to prove what is asked. Congruent sides, angles, etc. should all be marked so that you can see for yourself what must be written in the proof to convince the reader that you are right in your conclusion.
4. Then go to another given, and repeat. Finally, list the conclusion to be proved. Now you have a beginning and an end to the proof.

Note: Write the steps down carefully, without skipping even the simplest one. Some of the first steps are often the given statements (but not always), and the last step is the conclusion that you set out to prove.

Remember that algebra is about WHAT (finding the unknown) and geometry is about WHY. You must always explain *why*.

When you see PROVE or CONCLUDE, then you need to write a proof.

OPEN YOUR ASN:

Theorem 1 *If two angles are right angles, then they are congruent.*

Given: $\angle A$ is a right \angle .
 $\angle B$ is a right \angle .

Prove: $\angle A \cong \angle B$



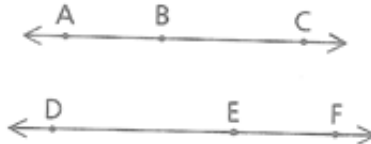
Proof:

Statements	Reasons
1 $\angle A$ is a right angle.	1 Given
2 $m\angle A = 90$	2
3 $\angle B$ is a right angle.	3 Given
4 $m\angle B = 90$	4
5 $\angle A \cong \angle B$	5 If two angles have the same measure, then they are congruent. (See steps 2 and 4.)

Theorem 2 *If two angles are straight angles, then they are congruent.*

Given: $\angle ABC$ is a straight angle.
 $\angle DEF$ is a straight angle.

Prove: $\angle ABC \cong \angle DEF$



Proof:

Statements	Reasons
1 $\angle ABC$ is a straight angle.	1 Given
2 $m\angle ABC = 180$	2
3 $\angle DEF$ is a straight angle.	3
4 $m\angle DEF = 180$	4
5 $\angle ABC \cong \angle DEF$	5

Do you see the theorem in the proof of itself?

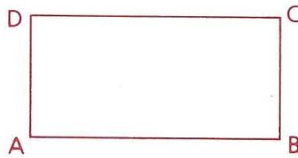
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One BIG mistake that students often make is to use sample problems in their system. A sample problem shows you how you use the theorem; it is NOT a proof of the theorem. You may NOT use a theorem to prove itself. That is circular logic (and it's wrong). It's like me telling you that the definition of a "widget" is a widget; you cannot use something to define or justify itself.

The sample problems are good practice though.

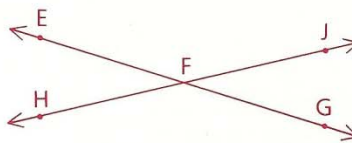
Part Two: Sample Problems

Problem 1 Given: $\angle A$ is a right angle.
 $\angle C$ is a right angle.
Conclusion: $\angle A \cong \angle C$



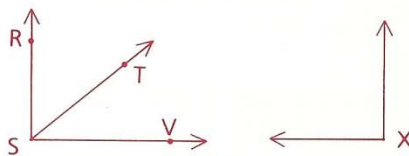
Statements	Reasons
1 $\angle A$ is a right angle.	1 Given
2 $\angle C$ is a right angle.	2 Given
3 $\angle A \cong \angle C$	3

Problem 2 Given: Diagram as shown
Conclusion: $\angle EFG \cong \angle HFJ$



Statements	Reasons
1 Diagram as shown	1 Given
2 $\angle EFG$ is a straight angle.	2 Assumed from diagram
3 $\angle HFJ$ is a straight angle.	3 Assumed from diagram
4 $\angle EFG \cong \angle HFJ$	4

Problem 3 Given: $\angle RST = 50^\circ$,
 $\angle TSV = 40^\circ$;
 $\angle X$ is a right angle.
Prove: $\angle RSV \cong \angle X$

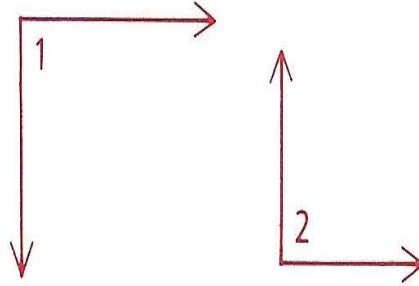


Statements	Reasons
1 $\angle RST = 50^\circ$	1 Given
2 $\angle TSV = 40^\circ$	2 Given
3 $\angle RSV = 90^\circ$	3 Addition ($50^\circ + 40^\circ = 90^\circ$)
4 $\angle RSV$ is a right angle.	4
5 $\angle X$ is a right angle.	5 Given
6 $\angle RSV \cong \angle X$	6

Problem Set A

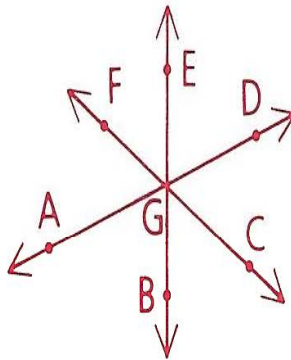
In problems 1 and 2, copy the figure and the incomplete proof. Then complete the proof by filling in the missing reasons.

- 1 Given: $\angle 1$ is a right \angle .
 $\angle 2$ is a right \angle .
 Prove: $\angle 1 \cong \angle 2$



Statements	Reasons
1 $\angle 1$ is a right angle.	1 _____
2 $\angle 2$ is a right angle.	2 _____
3 $\angle 1 \cong \angle 2$	3 _____

- 2 Given: Diagram as shown
 Prove: $\angle AGD \cong \angle EGB$

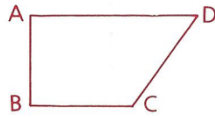


Statements	Reasons
1 Diagram as shown	1 _____
2 $\angle AGD$ is a straight angle.	2 _____
3 $\angle EGB$ is a straight angle.	3 _____
4 $\angle AGD \cong \angle EGB$	4 _____

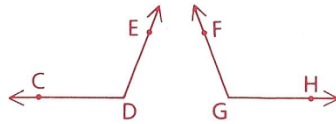
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In problems 3-7, use the two-column form of proof.

- 3 Given: $\angle A$ is a right angle.
 $\angle B$ is a right angle.
 Prove: $\angle A \cong \angle B$



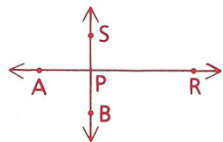
- 4 Given: $\angle CDE = 110^\circ$,
 $\angle FGH = 110^\circ$
 Conclusion: $\angle CDE \cong \angle FGH$



- 5 Given: $JK = 2.5$ cm, $NO = 2.5$ cm
 Conclusion: $\overline{JK} \cong \overline{NO}$

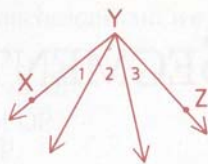


- 6 Given: Diagram as shown
 Prove: $\angle APR \cong \angle SPB$



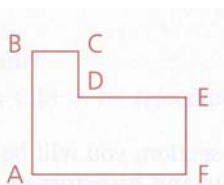
- 7 Given: $\angle 1 = 20^\circ$,
 $\angle 2 = 40^\circ$,
 $\angle 3 = 30^\circ$

Prove: $\angle XYZ$ is a right angle.

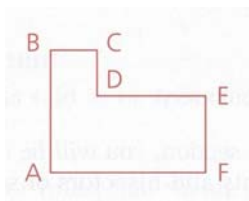


- 8 Draw the figure ABCDEF.

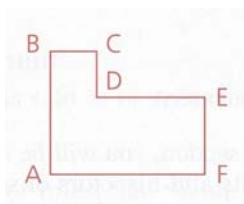
- a Draw its reflection over \overleftrightarrow{AF} .
 b Draw its reflection over \overleftrightarrow{AB} .
 c Draw a 90° clockwise rotation of the figure about B.



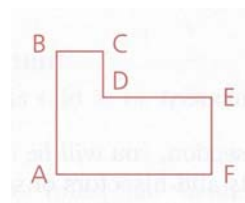
8a



8b



8c



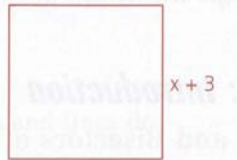
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9 Find the angle formed by the hands of a clock at 11:40.

10 The square has a perimeter of 42.

a Solve for x .

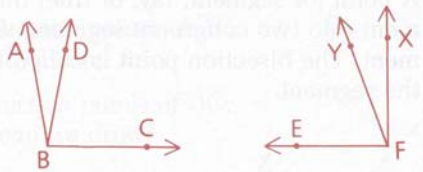
b If the perimeter were greater than 42, what would we know about the value of x ?



Problem Set B

11 Given: $\angle ABD = 10^\circ$,
 $\angle ABC = 100^\circ$,
 $\angle EFY = 70^\circ 20'$,
 $\angle XFY = 19^\circ 40'$

Prove: $\angle DBC \cong \angle XFE$



12 Point P has a coordinate of 7 on a number line. If you “slide” P 15 units in the negative direction, what are the coordinates of the resulting point P’?

13 a Draw a number line, labeling points A = (-1) and B = (5). Then label point A’, the reflection of A over B.

b Does $AB = BA'$?

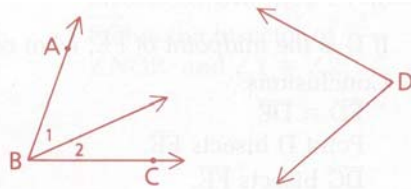
c What do we know about point B?

Problem Set C

14 The measure of an obtuse angle is $5y + 45$. What are the restrictions on y?

15 Given: $\angle 1 = (x + 7)^\circ$,
 $\angle 2 = (2x - 3)^\circ$,
 $\angle ABC = (x^2)^\circ$,
 $\angle D = (5x - 4)^\circ$

Show that $\angle ABC \cong \angle D$.



NOTE: SHOW is not PROVE.

1.4 Beginning Proof

Rewrite your “pretty proof” here. Math is not different than other subjects; you want to submit your best (edited) work. You need to rewrite the entire problem including the diagram, the given, and the prove statements. THEN, rewrite your two-column proof. If you do not do this before the next class, you will receive a zero (0) grade for the next assignment. You will not be given class time to complete this.