Pages 589 – 592 (Section 12.6)

1 a $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3^3) = \frac{4}{3}\pi(27)$

$V_{sphere} = 36\pi$

b $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9^3) = \frac{4}{3}\pi(729)$

$V_{sphere} = 927\pi$

c $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5^3) = \frac{4}{3}\pi(125)$

$V_{sphere} = \frac{500}{3}\pi$

2 $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = \frac{4}{3}\pi(216)$

$V_{sphere} = 288\pi$

$TA_{sphere} = 4\pi r^2 = 4\pi(6)^2 = 4\pi(36)$

$TA_{sphere} = 144\pi$

3 $V_{cyl} = B \cdot h$

$V_{cyl} = 9\pi(15) = 135\pi$

$V_{\frac{1}{2}sphere} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{1}{2}(\frac{4}{3}\pi)(27)$

$V = 18\pi$

$V_{silo} = 135\pi + 18\pi = 153\pi \approx 481$ cu m

4 a $V_{cyl} = Bh = \pi r^2 h$

$r = \frac{1}{2}(14) = 7$

$V_{cyl} = \pi(7)^2(8) = 392\pi$

b $V_{hemisphere} = \frac{1}{2}V_{sphere} = \frac{1}{2}(\frac{4}{3}\pi 6^3)$

$V_{hemisphere} = 144\pi$

c $V_{plastic} = V_{cyl} - V_{hemisphere}$

$V_{plastic} = 392\pi - 144\pi = 248\pi$

5 $V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5^3) = \frac{4}{3}\pi(125)$

$V_{sphere} = \frac{500}{3}\pi \approx (166)(3.14) \approx 523$ cu ft

6 $r = \frac{1}{2}(48) = 24$

$V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(24)^3 = \frac{4}{3}\pi(13824)$

$V_{sphere} = 18{,}432\pi$

$r = \frac{1}{2}(42) = 21$

$V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(21)^3 = \frac{4}{3}\pi(9261) = 12{,}348\pi$

$V_{rubber\ used} = 18{,}432\pi - 12{,}348\pi = 6084\pi \approx 19$ cu cm

7 a $V_{\frac{1}{2}sphere} = \frac{1}{2}(\frac{4}{3}\pi r^3)$

$V_{\frac{1}{2}sphere} = \frac{1}{2}(\frac{4}{3}\pi)(216) = 144\pi$

$V_{cone} = \frac{1}{3}Bh = \frac{1}{3}(36\pi)(8)$

$V_{cone} = 96\pi$

Total volume $= 144\pi + 96\pi = 240\pi$

b $A_{\frac{1}{2}sphere} = \frac{1}{2} \cdot 4\pi r^2$

$A_{\frac{1}{2}sphere} = \frac{1}{2}(4\pi)(36) = 72\pi$

$A_{cone} = \pi r \ell$ ℓ = slant height = 10

$A_{cone} = \pi \cdot 6 \cdot 10$ (Pythagorean triple 6-8-10)

$A_{cone} = 60\pi$

Total surface area $= 72\pi + 60\pi = 132\pi$

8 a $V_{hemisphere} = \frac{1}{2}V_{sphere} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi(30)^3 = \frac{2}{3}\pi(27{,}000)$

$V_{hemisphere} = 18{,}000\pi \approx 56{,}549$ cu m

b $A \odot = \pi r^2$

$A \odot = \pi(30)^2$

$A \odot = 900\pi \approx 2827$ sq m

c $A_{hemisphere} = \frac{1}{2}A_{sphere} = \frac{1}{2}(4\pi r^2) = 2\pi(30)^2$

$A_{hemisphere} = 1800\pi$ sq m, $A \odot = 900\pi$ sq m

Twice as much paint is needed to cover the area of the hemisphere.

d $1800\pi = \pi r^2$

$1800 = r^2$

$r = 30\sqrt{2} \approx 42$ m

9 The cold capsule is a cylinder and a sphere.

$V_{cyl} = B \cdot h$

$V_{cyl} = \frac{9}{4}\pi(8) = 18\pi$

$V_{sphere} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(\frac{3}{2})^3$

$V_{sphere} = \frac{4}{3}\pi(\frac{27}{8}) = \frac{9}{2}\pi$

Total Volume $= 18\pi + \frac{9}{2}\pi = 22\frac{1}{2}\pi \approx 71$ cu mm

10 a $V_{sphere} = \frac{4}{3}\pi r^3$ and the ratio of the radii is 2:5, so the ratio of the Volumes is $2^3:5^3$ or 8:125.

b $A_{sphere} = 4\pi r^2$ and the ratio of the radii is 2:5, so the ratio of the Areas is $2^2:5^2$ or 4:25.

11 a The hemisphere, cylinder, and cone all have a radius of 3.

$$V_{hemi} = \frac{1}{2}V_{sphere} \qquad V_{cyl} = B \cdot h$$
$$= \frac{1}{2}(\frac{4}{3}\pi r^3) \qquad = (\pi r^2)h$$
$$= \frac{2}{3}\pi(3)^3 \qquad = (\pi \cdot 3^2)12$$
$$V_{hemi} = 18\pi \qquad V_{cyl} = 108\pi$$
$$V_{cone} = \frac{1}{3}B \cdot h$$
$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi(3^2)4$$
$$V_{cone} = 12\pi$$
$$TA = V_{hemi} + V_{cyl} + V_{cone}$$
$$= 18\pi + 108\pi + 12\pi = 138\pi$$

b $LA_{cyl} = 2\pi rh$ $\qquad \ell = 5$ in a 3-4-5 rt $\triangle$.

$$LA_{cyl} = 2\pi 3(12) \qquad LA_{cone} = \pi r\ell$$
$$LA_{cyl} = 72\pi \qquad LA_{cone} = \pi(3)(5)$$
$$LA_{cone} = 15\pi$$
$$A_{hemisphere} = \frac{1}{2}(4\pi r^2)$$
$$A_{hemisphere} = 2\pi(3)^2$$
$$A_{hemisphere} = 18\pi$$

Total surface area = $72\pi + 15\pi + 18\pi = 105\pi$

12 The radius of ice cream and cone is 2.

$$V_{ice\ cream} = \frac{4}{3}\pi r^3 \qquad V_{cone} = \frac{1}{3}Bh$$
$$= \frac{4}{3}\pi(2)^3 \qquad = \frac{1}{3}(\pi r^2)h$$
$$= \frac{32}{3}\pi \qquad = \frac{1}{3}(\pi(2^2))9$$
$$V_{ice\ cream} = 10\frac{2}{3}\pi \qquad = \frac{1}{3}(36\pi)$$
$$V_{cone} = 12\pi$$

No. The cone will hold 12π cu cm and the ice cream is only $10\frac{2}{3}$ cu cm.

13 a The radius of the largest inscribed sphere is $\frac{1}{2}$ the side of the cube. Since $V_{cube} = s^3$ and $s^3 = 1000$, $s = 10$ and the radius is 5.

$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi 5^3$$
$$= \frac{4}{3}(125)\pi = \frac{500}{3}\pi \approx 524 \text{ cu m}$$

b The radius of the smallest circumscribed sphere is $\frac{1}{2}$ the diagonal of the cube. From a 45°45°90° $\triangle$, the diagonal is $10\sqrt{3}$ and the radius $= 5\sqrt{3}$.

$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi(5\sqrt{3})^3$$
$$= \frac{4}{3}\pi(375\sqrt{3}) = 500\pi\sqrt{3} \approx 2721 \text{ cu m}$$

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$$V_{sphere} = \frac{4}{3}\pi r^3$$
$$V_{cyl} = Bh$$
$$V_{cyl} = \pi r^2(2r)$$
$$V_{cyl} = 2\pi r^3$$
$$\frac{V_{sphere}}{V_{cyl}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$

15

$$A\odot = \pi r^2$$
$$= \pi \cdot 6^2$$
$$= 36\pi$$
$$A_{rect\ ABGH} = 1 \cdot 12 = 12$$
$$\therefore (\frac{12}{36\pi} \cdot 100)\% = \frac{100}{3\pi}\% \approx 11\%$$

16

$$V_{hemi} = \frac{1}{2}(\frac{4}{3}\pi r^3) = \frac{2}{3}\pi r^3$$
$$V_{cone} = \frac{1}{3}\pi r^2(r) = \frac{1}{3}\pi r^3$$
$$\frac{V_{hemi}}{V_{cone}} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^3} = \frac{2}{1}$$

17 The height of both prisms is the same, so by Cavalieri's principle, if $A_{shell} = A_{cyl}$, then $V_{shell} = V_{cyl}$.

$$A_{shell} = A_{cyl} - A_{inner\ cyl} \qquad A_{cyl} = \pi(r\sqrt{3})^2 h$$
$$A_{shell} = \pi(2r)^2 \cdot h - \pi r^2 \cdot h \qquad A_{cyl} = 3\pi r^2 h$$
$$A_{shell} = 3\pi r^2 h$$

18 a $A_{annulus} = \pi R^2 - \pi d^2$

The radius of the $\odot$ is $\sqrt{R^2 - d^2}$ by Pythagorean Theorem.

$A_{circle} = \pi(\sqrt{R^2 - d^2})^2$

$A_{circle} = \pi R^2 - \pi d^2$

b By Cavalieri's principle the volume of the hemisphere is equal to the volume of the cylinder minus the volume of the cone.

$V_{hemi} = V_{cyl} - V_{cone}$

$V_{hemi} = \pi R^2(R) - \frac{1}{3}\pi R^2(R)$

$V_{hemi} = \frac{2}{3}\pi R^3$

Therefore, the volume of a sphere $= \frac{4}{3}\pi R^3$.