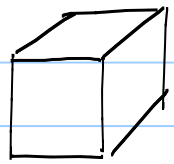
12-5 16, 15, 14, 13, 10, 9

Note Title 5/11/2016

16 Find the volume of a cube whose total surface area is 150 sq in.



$\frac{SA}{\text{faces}} = A \mid \text{face} \Rightarrow \frac{150}{6} = 25 \Rightarrow \text{side} = 5$

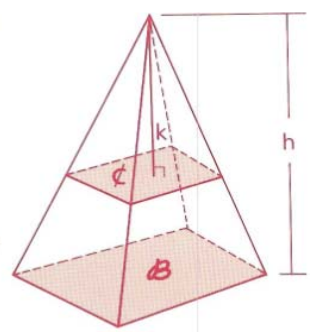
$V = 5^3 = 125$

15 Set up and complete a proof of Theorem 121. (Hint: First prove that the ratio of corresponding segments of a cross section and a base equals the ratio of h to k.)

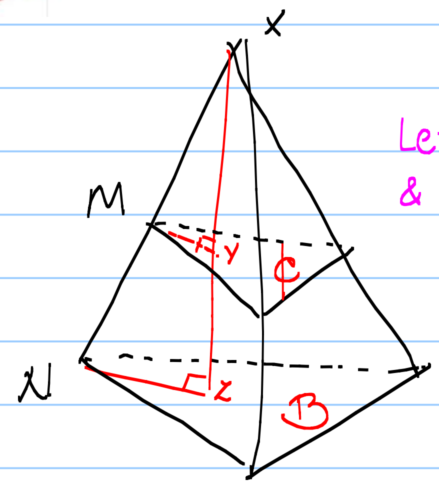
Theorem 121 *In a pyramid or a cone, the ratio of the area of a cross section to the area of the base equals the square of the ratio of the figures' respective distances from the vertex.*

$$\frac{\text{¢}}{\mathcal{B}} = \left(\frac{k}{h}\right)^2$$

where ¢ is the area of the cross section, $\mathcal{B}$ is the area of the base, k is the distance from the vertex to the cross section, and h is the height of the pyramid or cone.



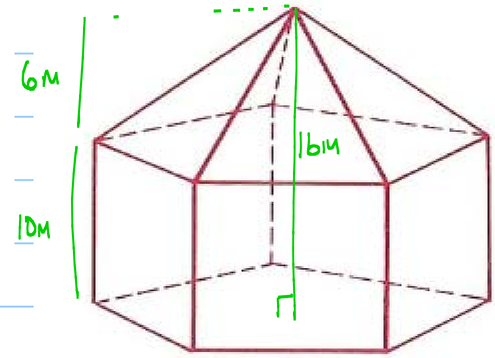
G: Diag P: $\frac{\text{¢}}{B} = \left(\frac{K}{h}\right)^2$



Let XY = K & XZ = h

1. Diag — 1. Given
2. ∠XYM & ∠XZN rt∠s — 2. Alt ⇒ rt∠
3. ∠XYM ≅ ∠XZN — 3. rt∠s ⇒ ≅∠s
4. ∠MXY ≅ ∠NXZ — 4. reflexive
5. △MXY ~ △NXZ — 5. AA~
6. $\frac{K}{h} = \frac{MY}{NZ}$ — 6. ~△s ⇒ corr sds prop.
7. $\left(\frac{K}{h}\right)^2 = \left(\frac{MY}{NZ}\right)^2$ — 7. ~ POLYS ⇒ ~ AREAS
8. $\left(\frac{MY}{ZN}\right)^2 = \frac{\text{¢}}{B}$ — 8. ~ FIGS ⇒ RATIO LENGTH^2 = RATIO AREAS
9. $\left(\frac{K}{h}\right)^2 = \frac{\text{¢}}{B}$ — 9. Substitution

13 A gazebo (garden house) has a pentagonal base with an area of 60 sq m. The total height to the peak is 16 m. The height of the pyramidal roof is 6 m. Find the gazebo's total volume.



$V_{pyr} + V_{PRISM}$

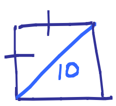
$\frac{1}{3}Bh \qquad B \cdot h$

$\frac{1}{3} 60 \cdot 6 + 60 \cdot 10$

$20 \cdot 6 +$

$120 + 600 = 720m^2$

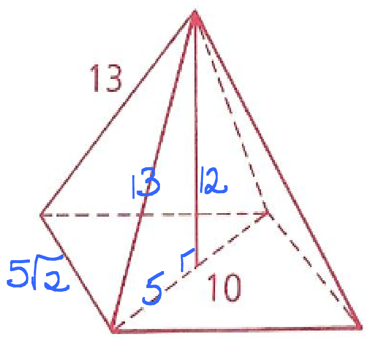
9 A pyramid has a square base with a diagonal of 10. Each lateral edge measures 13. Find the volume of the pyramid.



45 45 90

$x \quad x \quad x\sqrt{2}$

10



If $x\sqrt{2} = 10$

$x = \frac{10}{\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}}$ then $x = 5\sqrt{2}$

$V = \frac{1}{3} B \quad h$

$\frac{1}{3} (5\sqrt{2})^2 (12)$

$\frac{1}{3} 25 \cdot 2 \cdot 12$

$4 \cdot 25 \cdot 2$

$8 \cdot 25$

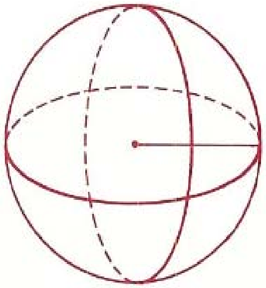
200

Note Title 5/11/2016

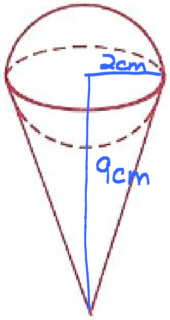
Theorem 122 *The volume of a sphere is equal to four thirds of the product of π and the cube of the radius.*

$$V_{sphere} = \frac{4}{3}\pi r^3$$

where r is the radius of the sphere.



12 An ice-cream cone is 9 cm deep and 4 cm across the top. A single scoop of ice cream, 4 cm in diameter, is placed on top. If the ice cream melts into the cone, will it overflow? (Assume that the ice cream's volume does not change as it melts.) Justify your answer.



V_{SPH}

$\frac{4}{3}\pi r^3$

$\frac{4}{3}\pi 2^3$

$\frac{32}{3}\pi$

$\approx 11\pi$

V_{CONE}

$\frac{1}{3}Bh$

$\frac{1}{3}\pi r^2 \cdot h$

$\frac{1}{3}\pi(2)^2 9$

12π

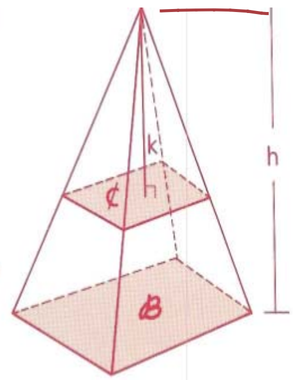
The I.C. will fit in the cone.
It will not overflow.

15 Set up and complete a proof of Theorem 121. (Hint: First prove that the ratio of corresponding segments of a cross section and a base equals the ratio of h to k.)

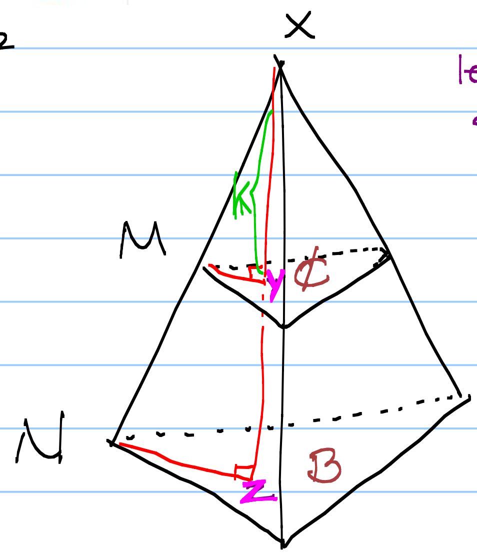
Theorem 121 *In a pyramid or a cone, the ratio of the area of a cross section to the area of the base equals the square of the ratio of the figures' respective distances from the vertex.*

$$\frac{\cent}{\mathcal{B}} = \left(\frac{k}{h}\right)^2$$

where ¢ is the area of the cross section, $\mathcal{B}$ is the area of the base, k is the distance from the vertex to the cross section, and h is the height of the pyramid or cone.



Given Diag, Prove: $\frac{\cent}{B} = \left(\frac{k}{h}\right)^2$



let XY = k
& XZ = h

1. Diag	1. Given
2. $\angle XYM$ & $\angle XZN$ rt∠s	2. Alt ⇒ rt∠s
3. $\angle XYM \cong \angle XZN$	3. rt∠s ⇒ ≅∠s
4. $\angle MXY \cong \angle NXZ$	4. reflexive
5. $\triangle MXY \sim \triangle NXZ$	5. AA~
6. $\frac{XY}{XZ} = \frac{MY}{NZ}$	6. ~△ ⇒ corr sds prop
7. $\frac{k}{h} = \frac{MY}{NZ}$	7. substitute
8. $\left(\frac{k}{h}\right)^2 = \left(\frac{MY}{NZ}\right)^2$	8. SQ
9. $\left(\frac{k}{h}\right)^2 = \frac{\cent}{B}$	9. ~Figs (p 544)

14 Use the diagram at the right to find

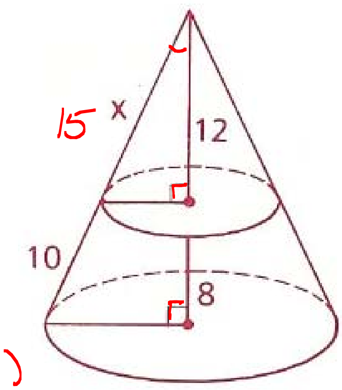
a x 15

b The radii of the circles 9 & 15

c The volume of the smaller cone

d The volume of the larger cone

e The volume of the frustum



~△ (AA~)

$\frac{x}{x+10} = \frac{3}{5}$, $5x = 3x + 30$

$2x = 30$

$x = 15$

$\frac{12}{20} \rightarrow \frac{3}{5}$

15 12 $r_1 = 9$

3(3 4 5) sm△

5(3 4 5) big△

c) $V = Bh/3$

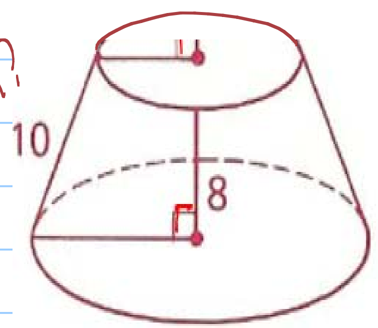
$\frac{1}{3}\pi r^2 h \rightarrow \frac{1}{3} 9^2 \cdot 12\pi = 324\pi$

d) $V = \frac{1}{3} Bh \rightarrow \frac{1}{3}\pi R h \rightarrow \frac{1}{3} 15^2 \cdot 25\pi$

$75 \cdot 20\pi = 1500\pi$

e

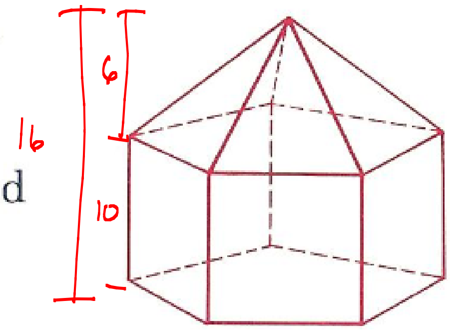
Test Question?



$V_{\text{LRG CONE}} - V_{\text{SM CONE}}$

$1500\pi - 324\pi = 1176\pi$

13 A gazebo (garden house) has a pentagonal base with an area of 60 sq m. The total height to the peak is 16 m. The height of the pyramidal roof is 6 m. Find the gazebo's total volume.



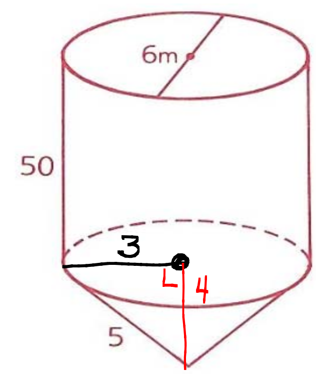
$$V_{PYR} + V_{PRISM}$$

$$\frac{1}{3}Bh + Bh$$

$$\frac{1}{3}60 \cdot 6 + 60 \cdot 10$$

$$120 + 600 = 720$$

7 A well has a cylindrical wall 50 m deep and a diameter of 6 m. The tapered bottom forms a cone with a slant height of 5 m. Find, to the nearest cubic foot, the volume of water the well could hold.



$$V_{CYL} + V_{CONE}$$

$$Bh + \frac{1}{3}Bh$$

$$\pi r^2 \cdot h + \frac{1}{3}\pi r^2 \cdot h$$

$$\pi 3^2 \cdot 50 + \frac{1}{3}\pi 3^2 \cdot 4$$

$$450\pi + 12\pi = 462\pi$$

462*π

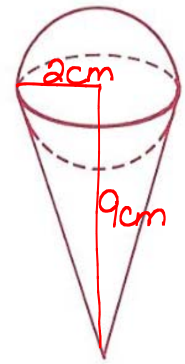
1451.415806

Moving On

$$SA_{SPHERE} = 4\pi r^2$$

$$V = \frac{1}{3}4\pi r^3 = \frac{4}{3}\pi r^3$$

12 An ice-cream cone is 9 cm deep and 4 cm across the top. A single scoop of ice cream, 4 cm in diameter, is placed on top. If the ice cream melts into the cone, will it overflow? (Assume that the ice cream's volume does not change as it melts.) Justify your answer.



Sphere

$\frac{4}{3}\pi r^3$

$\frac{4}{3}\pi 2^3$

$\frac{4}{3}\pi 8$

$\frac{32}{3}\pi$

Cone

$\frac{1}{3}Bh$

$\frac{1}{3}\pi r^2 (h)$

$\frac{1}{3}4\pi(9)$

$\frac{1}{3}36\pi$

12π

$\frac{32}{3}\pi < 12\pi$

NO

10 The radii of two spheres are in a ratio of 2:5.

a Find the ratio of their volumes. $(2/5)^3 = 8:125$

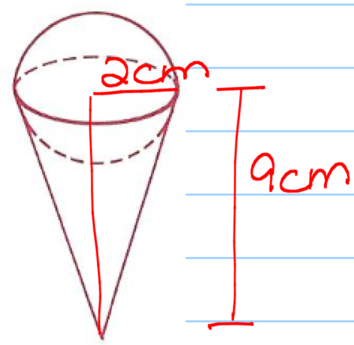
b Find the ratio of their surface areas. $(2:5)^2 = 4:25$

Q&A 12-5 15, 14, 9 see above

12.6 Prior knowledge $A_{SPH} = 4\pi r^2$

New : $V_{SPH} = \frac{4}{3}\pi r^3$

12 An ice-cream cone is 9 cm deep and 4 cm across the top. A single scoop of ice cream, 4 cm in diameter, is placed on top. If the ice cream melts into the cone, will it overflow? (Assume that the ice cream's volume does not change as it melts.) Justify your answer.



V_{sph}

$4/3\,\pi r^3$

$4/3\,\pi\, 8$

V_{cone}

$\frac{1}{3}Bh$

$\frac{1}{3}\pi r^2 h$

$\frac{32}{3}\pi < 12\pi$

No!

10 The radii of two spheres are in a ratio of 2:5.

a Find the ratio of their volumes.

b Find the ratio of their surface areas.

B $\left(\frac{2}{5}\right)^2 = \frac{4}{25}$

A $\left(\frac{2}{5}\right)^3 = \frac{8}{125}$

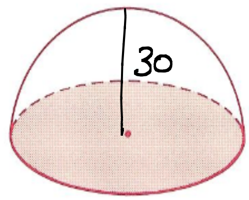
8 A hemispherical dome has a height of 30 m.

a Find, to the nearest cubic meter, the total volume enclosed.

b Find, to the nearest square meter, the area of ground covered by the dome (the shaded area).

c How much more paint is needed to paint the dome than to paint the floor?

d Find, to the nearest meter, the radius of a dome that covers double the area of ground covered by this one.



a) $\frac{1}{2}$ Sph

$\frac{1}{2} \cdot \frac{4}{3}\pi r^3$

$\frac{2}{3}\pi 30^3$

18000π = exact

est = 56549 cm^3

b) $\pi r^2 = A$

900π

2827