Note Title 4/25/2016

Objectives

After studying this section, you will be able to

- Find the areas of circles
- Find the areas of sectors
- Find the areas of segments

The Area of a Circle

You may already know the formula for the area of a circle.

Postulate ***The area of a circle is equal to the product of π and the square of the radius.***

$$A_{\odot} = \pi r^2$$

where r is the radius.

The Area of a Sector

The region bounded by a circle may be divided into ***sectors***.

Definition A ***sector*** of a circle is a region bounded by two radii and an arc of the circle.

Sector HOP

Just as the length of an arc is a fractional part of the circumference of a circle, the area of a sector is a fractional part of the area of the circle.

Theorem 108 ***The area of a sector of a circle is equal to the area of the circle times the fractional part of the circle determined by the sector's arc.***

$$A_{\text{sector HOP}} = \left(\frac{m\overset{\frown}{HP}}{360}\right)\pi r^2$$

where r is the radius and $\overset{\frown}{HP}$ is measured in degrees.

$\frac{\text{part}}{\text{whole}}$ (A)

The Area of a Segment

Another way of dividing the interior of a circle produces a ***segment***.

Definition A ***segment*** of a circle is a region bounded by a chord of the circle and its corresponding arc.

By studying the diagram above, you may be able to see what to do to find the area of a segment. Sample problem 4 will illustrate the procedure in detail.

Problem 1 *Find the area of a circle whose diameter is 10.*

$r = 5 \Rightarrow \pi r^2 = 25\pi$

Problem 2 *Find the circumference of a circle whose area is 49π sq units.*

$r = 7$

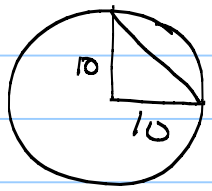
$\therefore C = \pi d$ or 14π

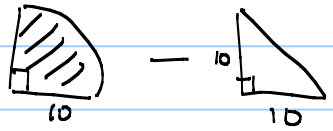
Problem 3 *Find the area of a sector with a radius of 12 and a 45° arc.*

$\frac{45}{360}\ \pi (12)^2$

$\frac{1}{8}\ \cancel{12}^{3} \cdot \cancel{12}^{6} \cdot \pi = 18\pi$

Problem 4 *The measure of the arc of the segment $(\overset{\frown}{AB})$ is 90. The radius of the circle is 10. Find the area of the segment.*





$\frac{90}{360}\ \pi 10^2 - \frac{1}{2}\ 10 \cdot 10$

$\frac{1}{4}\ 100\pi - \frac{1}{2}\ 100$

$25\pi - 50$