AMDG

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Adv Geo - 8 — 11.5: Areas of Regular Polygons — Date 22 Apr 16

Objectives

After studying this section, you will be able to

- Find the areas of equilateral triangles
- Find the areas of other regular polygons

Prior Knowledge

- Properties and areas triangles and quadrilaterals.

The Area of an Equilateral Triangle

Equilateral triangles are encountered so frequently that a special formula for their areas will be useful.

Remember that the altitude of an equilateral triangle divides it into two 30°-60°-90° right triangles.

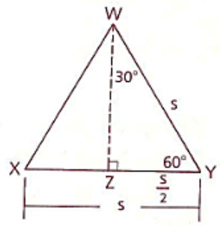
$x \quad x\sqrt{3} \quad 2x$

$\frac{s}{2} \quad \frac{s\sqrt{3}}{2}$

Thus, if WY = s, then ZY = $\frac{s}{2}$ and WZ = $\frac{s}{2}\sqrt{3}$.

Therefore, $A_{WXY} = \frac{1}{2}bh$

$= \frac{1}{2}s\left(\frac{s}{2}\sqrt{3}\right) = \frac{s^2}{4}\sqrt{3}$



$A = \frac{1}{2}\, b \cdot h$

$= \frac{1}{2}\; s \cdot \frac{s\sqrt{3}}{2}$

$= \frac{s^2\sqrt{3}}{4}$

Theorem 106 ***The area of an equilateral triangle equals the product of one-fourth the square of a side and the square root of 3.***

$$A_{eq.\,\Delta} = \frac{s^2}{4}\sqrt{3}$$

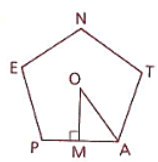
where s is the length of a side.

The Area of a Regular Polygon

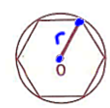
Recall that in a regular polygon all interior angles are congruent and all sides are congruent.

In regular polygon PENTA,

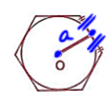
- O is the center
- $\overline{OA}$ is a *radius*
- $\overline{OM}$ is an *apothem*



Definition A *radius* of a regular polygon is a segment joining the center to any vertex.



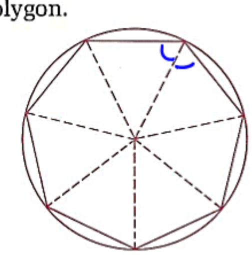
Definition An *apothem* of a regular polygon is a segment joining the center to the midpoint of any side.



Here are some important observations about apothems and radii:

- All apothems of a regular polygon are congruent.
- Only regular polygons have apothems.
- An apothem is a radius of a circle inscribed in the polygon.
- An apothem is the perpendicular bisector of a side.
- A radius of a regular polygon is a radius of a circle circumscribed about the polygon.
- A radius of a regular polygon bisects an angle of the polygon.

If all of the radii of a regular polygon are drawn, the polygon is divided into congruent isosceles triangles. (What is an altitude of each triangle?) If you write an expression for the sum of the areas of those isosceles triangles, you can derive the following formula.



Theorem 107 ***The area of a regular polygon equals one-half the product of the apothem and the perimeter.***

$$A_{reg.\,poly.} = \frac{1}{2}ap$$

where a is the length of an apothem and p is the perimeter.

Examples

Problem 1 *A regular polygon has a perimeter of 40 and an apothem of 5. Find the polygon's area.*

$A = \frac{1}{2} a \cdot p$

$= \frac{1}{2} \cdot 5 \cdot 40$

$= 100$

Problem 2 *An equilateral triangle has a side 10 cm long. Find the triangle's area.*

$A = \frac{s^2\sqrt{3}}{4}$

$= \frac{10^2\sqrt{3}}{4}$

$= 25\sqrt{3}$

Problem 3 *A circle with a radius of 6 is inscribed in an equilateral triangle. Find the area of the triangle.*

30	60	90
x	$x\sqrt{3}$	$2x$
6	$6\sqrt{3}$	

$s = 12\sqrt{3}$

$a = 6$

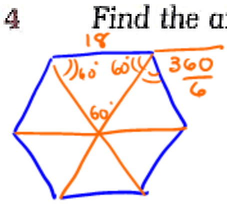
$A = \frac{1}{2} a \cdot p$

$\frac{1}{2}(6)(3 \cdot 12\sqrt{3})$

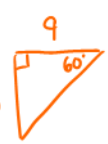
$108\sqrt{3}$

$A = \frac{s^2\sqrt{3}}{4} = \frac{(12\sqrt{3})^2\sqrt{3}}{4} = \frac{144 \cdot 3\sqrt{3}}{4} = 108\sqrt{3}$

Problem 4 *Find the area of a regular hexagon with sides 18 units long.*



" 6 △s



$a = 9\sqrt{3}$

$A = \frac{1}{2} a \cdot p$

$= \frac{1}{2}(9\sqrt{3})(6 \cdot 18)$

$= \frac{9 \cdot 6 \cdot 18}{2}\sqrt{3}$

$486\sqrt{3}$

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Homework

1 The perimeter of a regular polygon is 24 and the apothem is 3. Find the polygon's area.

2 Find the areas of equilateral triangles with the following sides.

a 6 **b** 7 **c** 8 **d** $2\sqrt{3}$

3 Find the areas of equilateral triangles with the following apothems.

a 6 **b** 4 **c** 3 **d** $2\sqrt{3}$

4 Find, to the nearest tenth, the area of a regular hexagon whose

a Side is 6

b Side is 8

c Apothem is 6

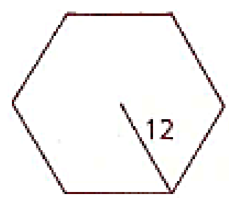
d Apothem is 8

5 The radius of a regular hexagon is 12.

Find: **a** The length of one side

b The apothem

c The area



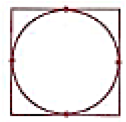
6 Find the area of a square whose

a Apothem is 5
b Apothem is 12
c Side is 7
d Diagonal is 10
e Radius is 6
f Perimeter is 12

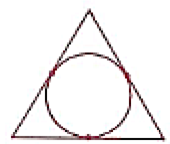
7 Find the apothem of a square whose area is 36 sq mm.

8 Find the side of an equilateral triangle whose area is $9\sqrt{3}$ sq km.

9 Find the area of a square if the radius of its inscribed circle is 9.



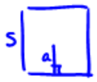
10 Find the area of an equilateral triangle if the radius of its inscribed circle is 3.



11 Find the area of a regular hexagon if the radius of its inscribed circle is 12.

Classwork

12 Find the area of



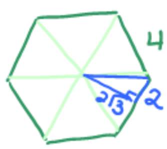
- **a** An equilateral triangle whose side is 9 $A = \frac{s^2\sqrt{3}}{4}$, $\frac{81}{4}\sqrt{3}$
- **b** A square whose apothem is $7\frac{1}{2}$ $\Rightarrow s = 15 \Rightarrow A = 225$
- **c** A regular hexagon whose side is 7

30 60 90
x $x\sqrt{3}$

$A = \frac{1}{2} a P$
$= \frac{1}{2} \cdot \frac{7\sqrt{3}}{2} \cdot 6 \cdot 7$
$\frac{7 \cdot 7 \cdot \cancel{6}^{3}}{2 \cdot \cancel{2}} \sqrt{3}$
$\frac{147}{2}\sqrt{3}$

13 Find the length of one side and of the apothem of

- **a** A square whose area is 121 $s = 11$, $a = 11/2$
- **b** An equilateral triangle whose area is $36\sqrt{3}$ sq m
- **c** A regular hexagon whose perimeter is 24 cm



$S = \frac{24}{6} = 4cm$
$a = 2\sqrt{3}cm$

$A = \frac{s^2\sqrt{3}}{4}$
$\frac{s^2\sqrt{3}}{4} = 36\sqrt{3}$
$\frac{s^2}{4} = 36$
$s^2 = 36 \cdot 4$
$s = 6 \cdot 2 = 12$
$a = 2\sqrt{3}$

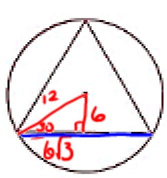
$\frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{6}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{3}}$
$\frac{6\sqrt{3}}{3} = a$

14 Find the perimeter of a regular polygon whose area is 64 and whose apothem is 4.

$A = \frac{1}{2} a P$
$\frac{64}{2} = \frac{\frac{1}{2} \cdot 4 \cdot P}{2}$
$32 = P$

15 A circle of radius 12 is circumscribed about each regular polygon below. Find the area of each polygon.

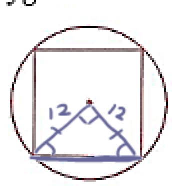
a



30 60 90
x $x\sqrt{3}$ 2x
12

$A = \frac{1}{2} a \cdot P$
$\frac{1}{2} 6 \cdot 3 \cdot 12\sqrt{3}$
$A = \frac{6 \cdot 3 \cdot \cancel{12}^{6}}{\cancel{2}}\sqrt{3}$
$A = 108\sqrt{3}$

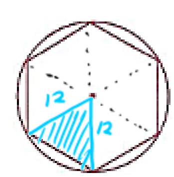
b



45 45 90
x x $x\sqrt{2}$
12 $12\sqrt{2}$
side

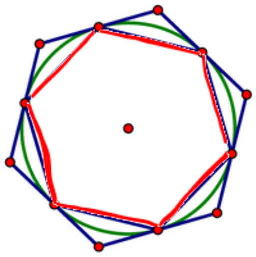
$A = (12\sqrt{2})^2$
$= 288$

c



$A = \frac{144\sqrt{3}}{4}$
$= 36\sqrt{3}$
$6 \cdot 36\sqrt{3}$
$216\sqrt{3}$

16 A circle is inscribed in one regular hexagon and circumscribed about another. If the circle has a radius of 6, find the ratio of the area of the smaller hexagon to the area of the larger hexagon.



$A = \frac{1}{2} a\,p$

$= \frac{1}{2}\, 3\sqrt{3} \cdot 6 \cdot 6$

30   60   90

$x\sqrt{3}$

6

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{6}{\sqrt{3}}$

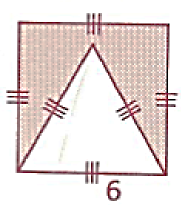
$x = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$

$A = \frac{1}{2}(6)(6)(4\sqrt{3})$

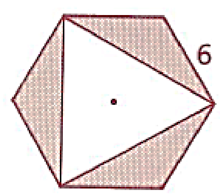
$\frac{\cancel{\frac{1}{2}}\, 3\cancel{\sqrt{3}} \cdot \cancel{6} \cdot \cancel{6}}{\cancel{\frac{1}{2}}\, \cancel{6} \cdot \cancel{6} \cdot 4\cancel{\sqrt{3}}} = \boxed{3:4}$

17 Find the area of the shaded region in each polygon. (Assume regular polygons.)

a



b



c