

AREAS OF PARALLELOGRAMS AND TRIANGLES

PRIOR KNOWLEDGE:

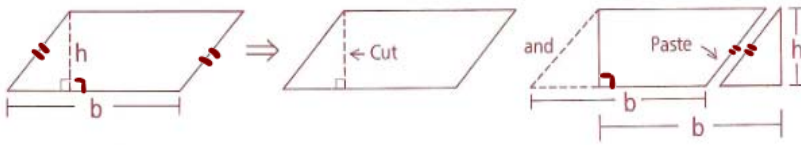
$A_{RECT} = l \cdot w \text{ or } b \cdot h$

Objectives

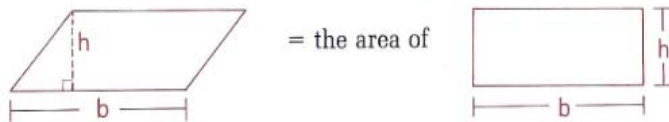
- After studying this section, you will be able to
- Find the areas of parallelograms
 - Find the areas of triangles

The Area of a Parallelogram

Many areas can be found by a "cut and paste" method. For example, to find the area of a parallelogram with base b and altitude h , we may do this:



Thus, the area of



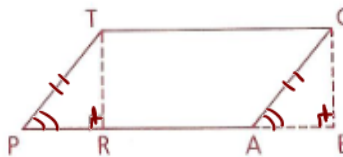
= the area of

Theorem 100 The area of a parallelogram is equal to the product of the base and the height.

$A = bh$

where b is the length of the base and h is the height.

Given: PACT is a \square , \overline{TR} & $\overline{CE} \perp \overline{PA}$



Prove: $A_{PACT} = (PA)(RT)$

- | | |
|--|---|
| 1. $\square PACT, \overline{TR} \text{ \& } \overline{CE} \perp \overline{PA}$ | 1. GIVEN |
| 2. $\angle PRT \text{ \& } \angle AEC \text{ r.t. } \angle$ s | 2. $\perp \Rightarrow \text{r.t. } \angle$ s |
| 3. $\angle PRT \cong \angle AEC$ | 3. r.t. \angle s $\Rightarrow \cong \angle$ s |
| 4. $\overline{TP} \parallel \overline{CA}$ | 4. $\square \Rightarrow \text{opp. sides } \parallel$ |
| 5. $\angle TPR \cong \angle CAE$ | 5. $\parallel \Rightarrow \text{corresponding } \angle$ s \cong |
| 6. $\overline{TP} \cong \overline{CA}$ | 6. $\square \Rightarrow \text{opp. sides } \cong$ |
| 7. $\triangle TPR \cong \triangle CAE$ | 7. AAS (3, 5, 6) |
| 8. $A_{\triangle TPR} = A_{\triangle CAE}$ | 8. $\cong \triangle \Rightarrow = \text{AREAS}$ |
| 9. $A_{TRAC} = A_{TRAC}$ | 9. REFLEXIVE |
| 10. $A_{TPAC} = A_{TREC}$ | 10. ADD |
| 11. $A_{PACT} = (RE)(TR)$ | 11. SUBSTITUTE |
| 12. $A_{PACT} = PA \cdot TR$ | 12. SUBSTITUTE |

$\cong \triangle$ s: ~~SSS~~

~~ASA~~

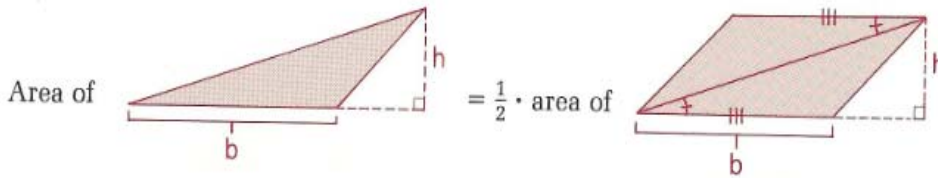
~~SAS~~

~~HL~~

AAS

The Area of a Triangle

The area of any triangle can be shown to be one half of the area of a parallelogram with the same base and height.

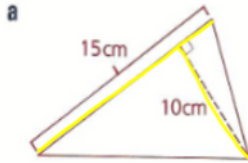


Theorem 101 *The area of a triangle is equal to one-half the product of a base and the height (or altitude) for that base.*

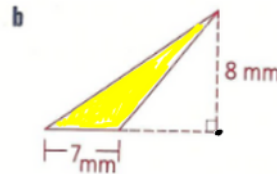
$$A_{\Delta} = \frac{1}{2}bh$$

where b is the length of the base and h is the altitude.

Problem 1 Find the area of each triangle.



$$A = \frac{1}{2}(15)(10) = 75$$



$$A = \frac{1}{2}(8)(7) = 28$$

Problem 2 Find the base of a triangle with altitude 15 and area 60.

$$A_{\Delta} = \frac{1}{2} b \cdot h$$

$$\frac{60}{15} = \frac{1}{2} b \cdot \frac{15}{15}$$

$$2 \cdot 4 = \frac{1}{2} b \cdot 2$$

$$8 = b$$

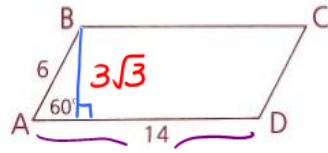
Problem 3 Find the area of a parallelogram whose sides are 14 and 6 and whose acute angle is 60° .

DROP ALTITUDE



$$\begin{array}{ccc} 30-60-90 & & \\ x & x\sqrt{3} & 2x \\ 3 & 3\sqrt{3} & 6 \end{array}$$

IF $2x=6$ THEN $x=3$
SUCH THAT ALT = $3\sqrt{3}$



$$\begin{aligned} A_{ABCD} &= b \cdot h \\ &= (14)(3\sqrt{3}) = (14 \cdot 3)\sqrt{3} = 42\sqrt{3} \end{aligned}$$

Problem 4 Find the area of a trapezoid WXYZ.

Using 11.1 } Sum of parts
& 11.2 }

I: $WB = 5 \therefore \Delta XWB = 5, 12, 13$

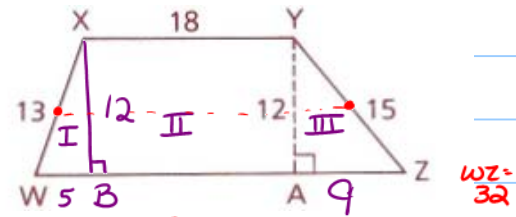
$$\therefore A_{\Delta XWB} = \frac{1}{2} \cdot 5 \cdot 12 = 30$$

II: $A_{ABXY} = 18(12) = 216$

III: $3(3 \cdot 4 \cdot 5)$

$$A_{\Delta AZY} = \frac{1}{2} \cdot 9 \cdot 12 = 54$$

$$\underline{\underline{300}}$$



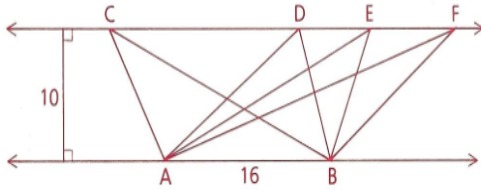
Using 11.3:

$$\begin{aligned} A_{\text{TRAPEZOID}} &= Mh \\ &= \left(\frac{b_1 + b_2}{2} \right) h \\ &= \left(\frac{XY + WZ}{2} \right) \cdot AY \\ &= \frac{18 + 32}{2} \cdot 12 \\ &= 25 \cdot 12 \\ &= 300 \end{aligned}$$

Homework: 1-25

(14: completed in large group)

- 14 Lines \overleftrightarrow{CF} and \overleftrightarrow{AB} are parallel and 10 mm apart. Several triangles with base \overline{AB} and a vertex on \overleftrightarrow{CF} have been drawn below. Which triangle has the largest area? Explain.



$$A_{\Delta} = \frac{1}{2} b \cdot h$$

$$A_{ABC} = \frac{1}{2} 16 \cdot 10$$

$$A_{ABD} = \frac{1}{2} 16 \cdot 10$$

$$A_{ABE} = \frac{1}{2} 16 \cdot 10$$

$$A_{ABF} = \frac{1}{2} 16 \cdot 10$$

ALL THESE AREAS ARE EQUAL \because SAME BASE (AB)
& $\overleftrightarrow{CF} \parallel \overleftrightarrow{AB}$. $\parallel \Rightarrow$ EQUIDIST. \therefore ALL HEIGHTS ARE =.