

### Objectives

- Understand the concept of area
- Find the areas of rectangles and squares
- Use the basic properties of area

### Prior Knowledge and Future Paths

- Definition of a function: For every  $x$ , there is one and only one  $y$ .

Example:

	Examples of a function	Examples that are not functions																																														
Linguistic:	Your bank balance is a function of time. Time (a continuous variable) is the independent variable, the $x$ . Balance (in Dollars) is the dependent variable, the $y$ .	The graph of a circle on the Cartesian plane is not a function																																														
Algebraic:	$y = \begin{cases} 50 & 0 \leq x < 2 \\ 55 & 2 \leq x < 4 \\ 15 & x > 4 \end{cases}$	$y^2 = 16 - x^2$																																														
Geometric (visual):																																																
Tabular:	<table border="1"> <thead> <tr> <th>Time (day)</th> <th>Balance (dollars)</th> </tr> </thead> <tbody> <tr><td>0</td><td>50</td></tr> <tr><td>1</td><td>50</td></tr> <tr><td>2</td><td>55</td></tr> <tr><td>3</td><td>55</td></tr> <tr><td>4</td><td>55</td></tr> <tr><td>5</td><td>15</td></tr> <tr><td>6</td><td>15</td></tr> <tr><td>7</td><td>15</td></tr> <tr><td>8</td><td>15</td></tr> <tr><td>9</td><td>15</td></tr> </tbody> </table>	Time (day)	Balance (dollars)	0	50	1	50	2	55	3	55	4	55	5	15	6	15	7	15	8	15	9	15	<table border="1"> <thead> <tr> <th>X</th> <th>Y1</th> <th>Y2</th> </tr> </thead> <tbody> <tr><td>0</td><td>-4</td><td>4</td></tr> <tr><td>1</td><td>-3.873</td><td>3.873</td></tr> <tr><td>2</td><td>-3.464</td><td>3.4641</td></tr> <tr><td>3</td><td>-2.646</td><td>2.6458</td></tr> <tr><td>4</td><td>0</td><td>0</td></tr> <tr><td>5</td><td>ERROR</td><td>ERROR</td></tr> <tr><td>6</td><td>ERROR</td><td>ERROR</td></tr> </tbody> </table>	X	Y1	Y2	0	-4	4	1	-3.873	3.873	2	-3.464	3.4641	3	-2.646	2.6458	4	0	0	5	ERROR	ERROR	6	ERROR	ERROR
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*Handwritten red arrows pointing from the left towards the 'Time and Money' graph.*

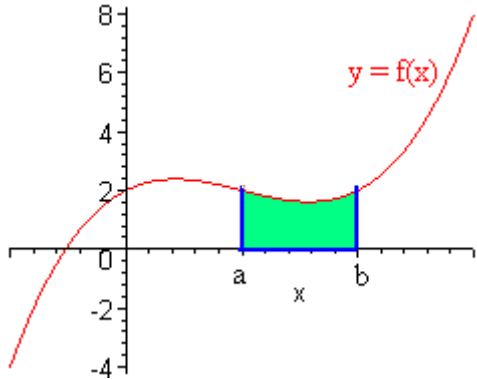
*Vertical Line Test*

*not a function*

not tested on this

- The First Fundamental Theorem of Calculus

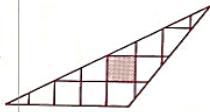
Let  $f(x)$  be a continuous positive function between  $a$  and  $b$  and consider the region below the curve  $y = f(x)$ , above the  $x$ -axis and between the vertical lines  $x = a$  and  $x = b$  as in the picture below.



We are interested in finding the area of this region.

**Definition** The area of a closed region is the number of square units of space within the boundary of the region.

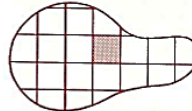
We can estimate the area of a region by determining the approximate number of square units it would take to fill the region.



Estimated Area = 10 sq units



Estimated Area = 18 sq units



Estimated Area = 19 sq units



Counting squares, however, is neither the easiest nor the best way to find the area of a region. We will develop formulas for computing the areas of regions bounded by the common geometrical figures. Such regions are usually named by their boundaries, as when we speak of "the area of a rectangle."

\*

**Postulate** The area of a rectangle is equal to the product of the base and the height for that base.  
 $A_{rect} = bh$   
 where  $b$  is the length of the base and  $h$  is the height.

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AMDG  
11: Area

11.1: Understanding Area

Ms. Kresovic  
Date \_\_\_\_\_

**Theorem 99** *The area of a square is equal to the square of a side.*  
$$A_{sq} = s^2$$
*where  $s$  is the length of a side.*

**Basic Properties of Area**

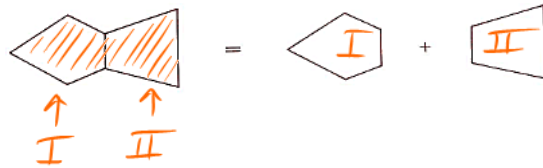
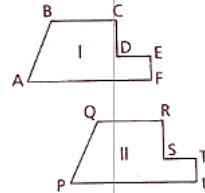
We make three basic assumptions about area:

**Postulate** *Every closed region has an area.*

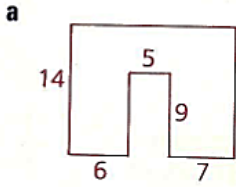
**Postulate** *If two closed figures are congruent, then their areas are equal.*

If  $ABCDEF \cong PQRSTU$ , then the area of region I = the area of region II.

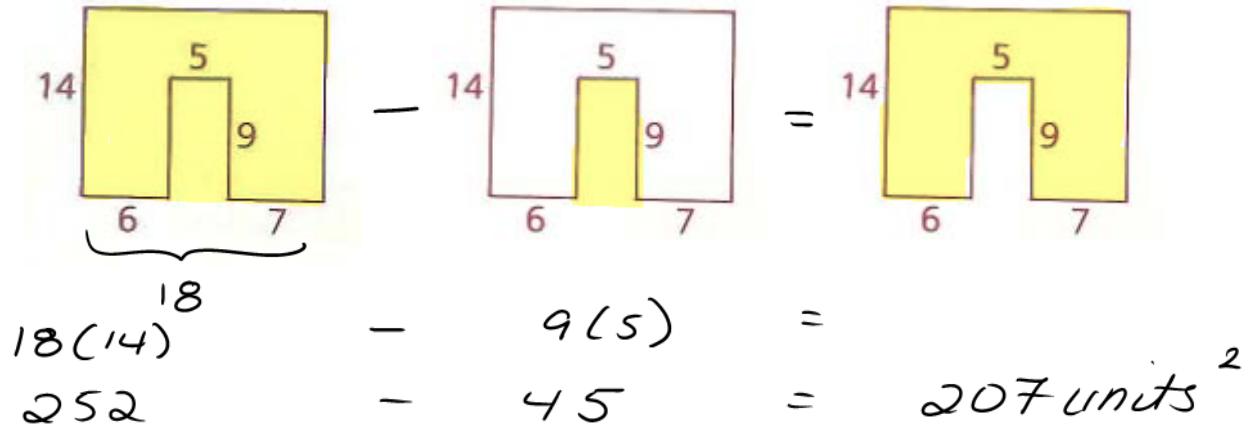
**Postulate** *If two closed regions intersect only along a common boundary, then the area of their union is equal to the sum of their individual areas.*



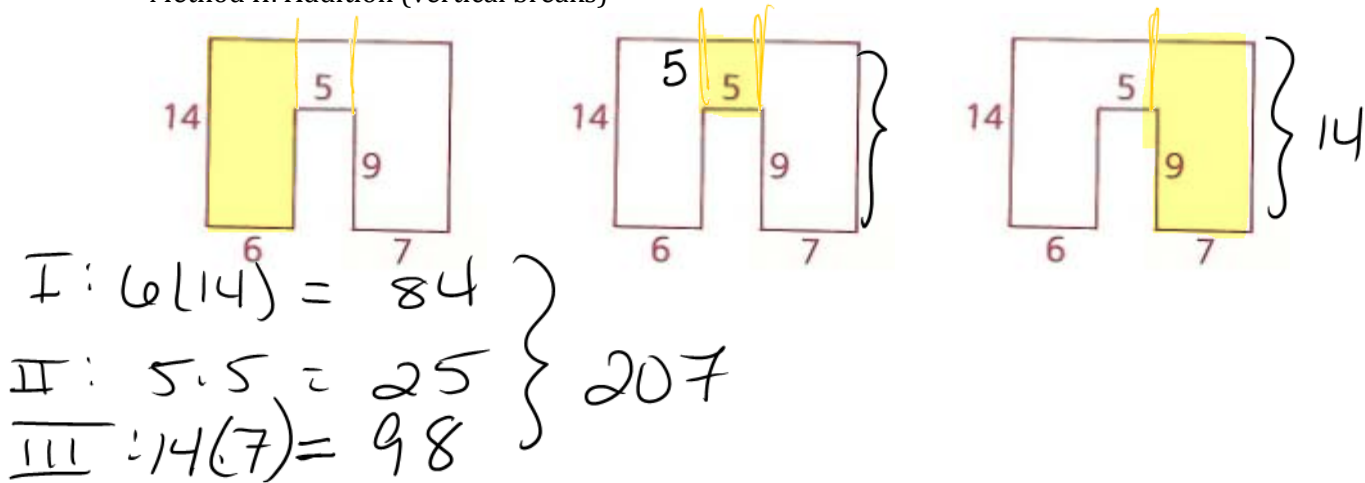
1 Find the area of each figure below. (Assume right angles.)



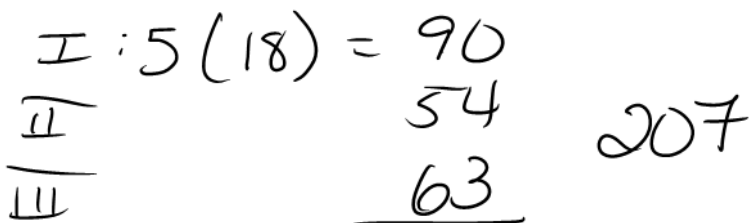
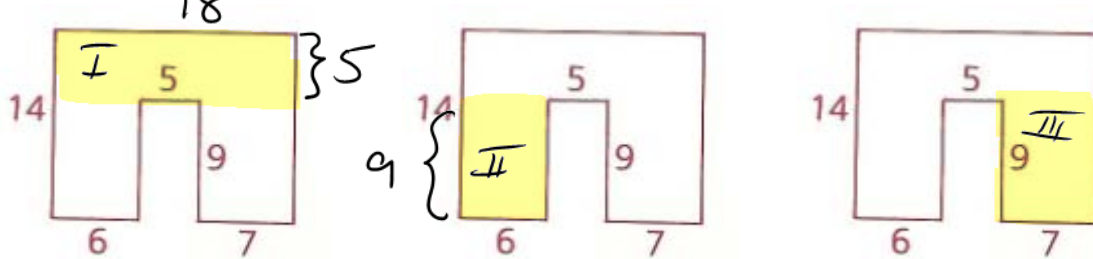
Method I: Subtraction



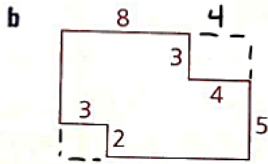
Method II: Addition (vertical breaks)



Method III: Addition (horizontal breaks)

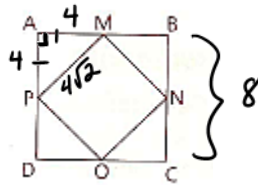


1 Find the area of each figure below. (Assume right angles.)



$$\begin{array}{r}
 \text{8} \\
 \text{12} \\
 \hline
 8(12) \\
 96
 \end{array}
 -
 \begin{array}{r}
 \text{3} \\
 \text{4} \\
 \hline
 3(4) \\
 12
 \end{array}
 -
 \begin{array}{r}
 \text{2} \\
 \text{3} \\
 \hline
 2(3) \\
 6
 \end{array}
 = 78$$

12 The area of square ABCD is 64 square units. MNOP is formed by joining the midpoints of the sides of ABCD. Find the area and the perimeter of MNOP.



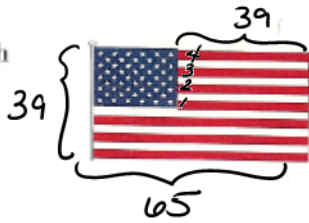
$$\begin{aligned}
 A &= 64 \Rightarrow s = 8 \\
 \text{midpt} &\Rightarrow \text{bis} \Rightarrow 4
 \end{aligned}$$

$$\begin{array}{r}
 45 - 45 - 90 \\
 \times \quad \times \quad \times \sqrt{2} \\
 4 \qquad 4\sqrt{2} = PM
 \end{array}$$

$$P_{MNOP} = 4(4\sqrt{2}) = 16\sqrt{2} \text{ units}$$

$$A_{MNOP} = (4\sqrt{2})^2 = 16 \cdot 2 = 32 \text{ units}^2$$

- 17 A flag has dimensions 65 by 39. Each short stripe has a length of 39. What fractional part of the flag is red?



EACH STRIPE:

$$\frac{39}{13} = 3$$

$$\text{whole: } 65(39) = 2535$$

$$\text{Red: } 4 \text{ short stripes: } (3 \cdot 39)$$

$$4 \cdot 3 \cdot 39 = 468$$

$$3 \text{ long stripes: } 3(65)$$

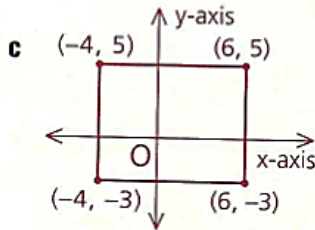
$$3 \cdot 3 \cdot 65 = 585$$

TTL  
RED  
AREA =  
1053

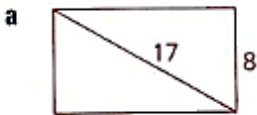
$$\frac{\text{Red}}{\text{whole}} = \frac{1053}{2535} = \frac{27}{65}$$

**Homework**

1 Find the area of each figure below. (Assume right angles.)



3 Find the area of each rectangle.



b

The perimeter is 40  
 One side is 6

4 The area of a rectangle is 48 sq mm, and the altitude is 6 mm.

- a Find the length of the base.
- b Find the length of a diagonal of the rectangle.

5 a Find the area of a square whose side is 12.

b Find the area of a square whose diagonal is 10.  $(5\sqrt{2})^2 = 50$

c Find the side of a square whose area is 49.

d Find the perimeter of a square whose area is 81.

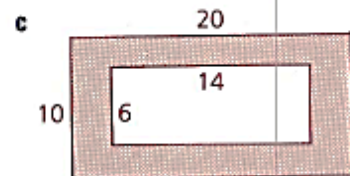
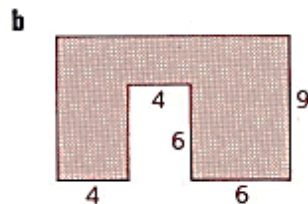
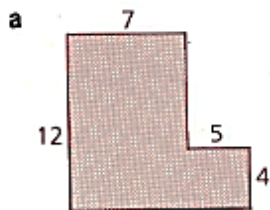
e Find the area of a square whose perimeter is 36.

5b:

$$\begin{matrix} 45 & 45 & 90 \\ \times & \times & \times \sqrt{2} \\ & & 10 \end{matrix}$$

$$1 \times \frac{10}{\sqrt{2}} = \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2} = 5$$

6 Find the area of each shaded region. (Assume right angles.)



7 The diagonal of a rectangle is  $\sqrt{29}$ , and the rectangle's base is 2.

a Find the area of the rectangle.  $2(5) = 10$

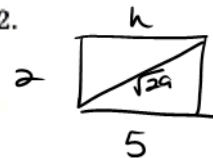
b Find its semiperimeter.  $2+5 = 7$   
 $\frac{1}{2}$  perimeter

$$2^2 + h^2 = (\sqrt{29})^2$$

$$h^2 = 29 - 4$$

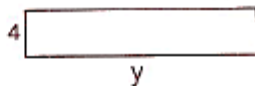
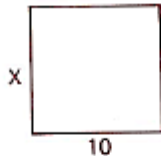
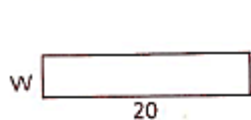
$$h^2 = 25$$

$$h = 5$$



### Problem Set B

8 Each rectangular garden below has an area of 100.



a Find the missing dimension of each.

b What length of fencing is needed to surround each?

c Which figure has the shortest perimeter?

d What do you think must be true about a rectangle that encloses the maximum possible area with the shortest possible perimeter?



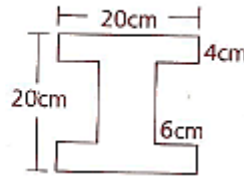
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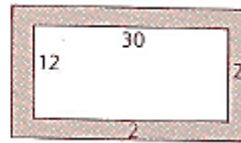
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Date \_\_\_\_\_

**Classwork**

- 9 A cross section of a steel I-beam is shown. Assume right angles and symmetry from appearances. Find the area of the cross section.



- 10 A rectangular picture measures 12 cm by 30 cm. It is mounted in a frame 2 cm wide. Find the area of the frame.



- 13 If the area of rectangle RCTN is six times the area of rectangle AECT, find the coordinates of A.

