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Adv Geo period _____ 10.7: Inscribed & Circumscribed Polygons Date________________

Objectives

After studying this section, you will be able to

- Recognize inscribed and circumscribed polygons
- Apply the relationship between opposite angles of an inscribed quadrilateral
- Identify the characteristics of an inscribed parallelogram

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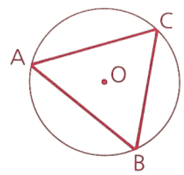
Part One: Introduction

Inscribed and Circumscribed Polygons

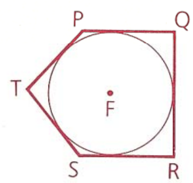
Triangle ABC is ***inscribed in*** circle O.

Definition A polygon is ***inscribed in*** a circle if all of its vertices lie on the circle.



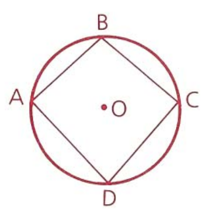
Polygon PQRST is ***circumscribed about*** circle F.

Definition A polygon is ***circumscribed about*** a circle if each of its sides is tangent to the circle.



We can also speak of a circle being circumscribed about a polygon or inscribed in a polygon.

The diagram shows that the statements "quadrilateral ABCD is inscribed in ⊙O" and "⊙O is circumscribed about quadrilateral ABCD" have the same meaning.



Definition The center of a circle circumscribed about a polygon is the ***circumcenter*** of the polygon.

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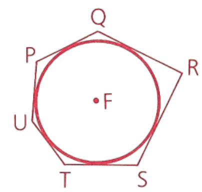
In the preceding diagram, O is the circumcenter of ABCD. Hexagon PQRSTU is circumscribed about circle F. Circle F is inscribed in hexagon PQRSTU.



Definition The center of a circle inscribed in a polygon is the ***incenter*** of the polygon.

F is the incenter of hexagon PQRSTU.

A Theorem About Inscribed Quadrilaterals

The following theorem can easily be proved by using the relationship between an inscribed angle and its intercepted arc.

Theorem 93 ***If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.***

Given: Quadrilateral ABCD is inscribed in circle O.

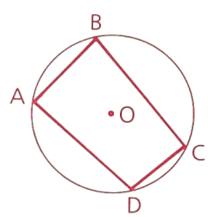
Prove: ∠A supp. ∠C, ∠B supp. ∠D

Proof: ∠A, ∠B, ∠C, and ∠D are inscribed angles, so

$m\angle A = \frac{1}{2}(m\overarc{BCD})$ and $m\angle C = \frac{1}{2}(m\overarc{BAD})$.

$$\begin{aligned} m\angle A + m\angle C &= \tfrac{1}{2}(m\overarc{BCD}) + \tfrac{1}{2}(m\overarc{BAD}) \\ &= \tfrac{1}{2}(m\overarc{BCD} + m\overarc{BAD}) \\ &= \tfrac{1}{2}(360) \qquad (\overarc{BCD} \cup \overarc{BAD} = \text{whole } \odot) \\ &= 180 \end{aligned}$$

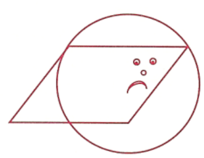
Thus, ∠A is supplementary to ∠C. Similarly, ∠B is supplementary to ∠D.

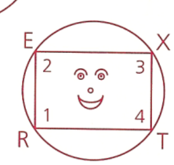


The Story of the Plain Old Parallelogram

Once there was a plain old parallelogram named Rex Tangle. Rex was always trying to fit in—into a circle, that is. One day when he awoke, he found that he had straightened out and was finally able to inscribe himself. What had the plain old parallelogram turned into?

The following theorem shows the moral of our story.





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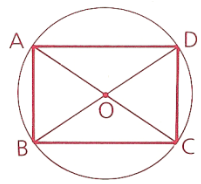
Theorem 94 ***If a parallelogram is inscribed in a circle, it must be a rectangle.***

Here are some of the conclusions that follow from Theorem 94.

If ABCD is an inscribed parallelogram, then

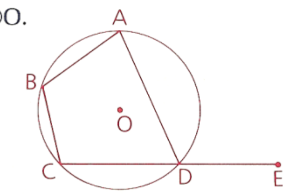
1. $\overline{BD}$ and $\overline{AC}$ are diameters
2. O is the center of the circle
3. $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, and $\overline{OD}$ are radii
4. $(AB)^2 + (BC)^2 = (AC)^2$, and so forth



Part Two: Sample Problems

Problem 1 *Given: Quadrilateral ABCD is inscribed in ⊙O.*

Prove: ∠B ≅ ∠ADE



Proof

1 ABCD is inscribed in ⊙O.	1 Given
2 ∠B supp. ∠ADC	2 If a quadrilateral is inscribed in a ⊙, its opposite ∠s are supp.
3 ∠ADC supp. ∠ADE	3 Two ∠s forming a straight ∠ are supp.
4 ∠B ≅ ∠ADE	4 Two ∠s supp. to the same ∠ are ≅.

Problem 2 *Parallelogram ABCD is inscribed in a circle, and its diagonals intersect at E.*

a *Draw the figure.*

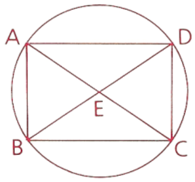
b *What is true about ▱ABCD?*

c *What is* $\overline{BD}$?

d *If AB = 5 and BC = 6, find AC.*

Solution

a



b A ▱ inscribed in a ⊙ must be a rectangle, so ABCD is a rectangle.

c ∠BCD is an inscribed right ∠, so $\frac{1}{2}(m\overarc{BAD}) = 90$, making $\overarc{BAD} = 180°$, a semicircle. Thus, $\overline{BD}$ is a diameter.

d Since △ABC is a right △,

$$(AB)^2 + (BC)^2 = (AC)^2$$
$$5^2 + 6^2 = (AC)^2$$
$$\sqrt{61} = AC$$

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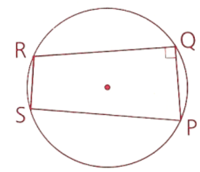
10.7 Class Examples

10 PQ = 15, QR = 20, RS = 7, and ∠Q is a right angle. Find PS.



If a quadrilateral is inscribed in a circle, then the opposite angles are

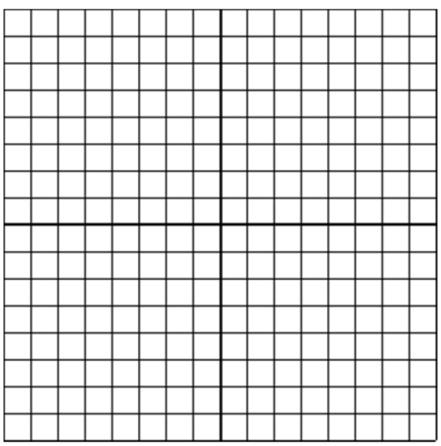
__________________________.

Therefore, m∠S = _____.

Mark the diagram and label the given lengths.

12 A circle is inscribed in a square with vertices (−8, −3), (−1, −3), (−8, 4), and (−1, 4).

- **a** Find the coordinates of the center of the circle.
- **b** Find the area of the circle.
- **c** Find the radius of a circle circumscribed about the square.



(a)

(b) Add a neat sketch to the diagram

What is the length of the side of the square? __________________

What is the radius of the circle? _________________________________

What is the formula for the area of a circle? ____________________

What is the area of this circle? ___________________________________

(c) Add a neat sketch to the diagram.

What is the radius of this circle? ________________________________

What is the area of this circle?___________________________________

14 Parallelogram RECT is inscribed in circle O. If RE = 6 and EC = 8, find the perimeter of △ECO.

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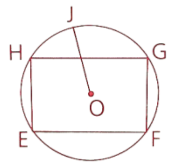
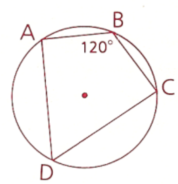
16 Given: ⊙O; EFGH is a ▱.
$\widehat{HG} = 120°$, OJ = 6
Find: The perimeter of EFGH



20 ABCD is a kite, with $\overline{AB} \cong \overline{BC}$, $\overline{AD} \cong \overline{CD}$, and m∠B = 120. The radius of the circle is 3. Find the perimeter of ABCD.



Problem Set C

21 Discuss the location of the center of a circle circumscribed about each of the following types of triangles.

a Right **b** Acute **c** Obtuse

22 A set of points are *concyclic* if they all lie on the same circle. Prove that the vertices of any triangle are concyclic.

23 Are the vertices of each figure concyclic Always, Sometimes, or Never?

a A rectangle
b A parallelogram
c A rhombus
d A nonisosceles trapezoid
e An equilateral polygon
f An equiangular polygon

24 A right triangle has legs measuring 5 and 12. Find the ratio of the area of the inscribed circle to the area of the circumscribed circle.

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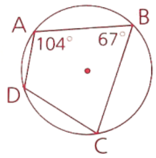
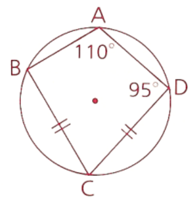
Homework

1 Given: $\angle A = 104°$, $\angle B = 67°$
Find: $\angle D$ and $\angle C$



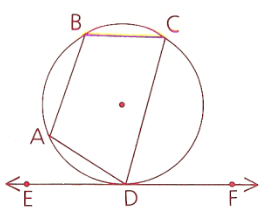
3 Given: $\angle A = 110°$, $\overline{BC} \cong \overline{CD}$, $\angle D = 95°$
Find: **a** $\angle C$ **c** $\angle B$
b $\overset{\frown}{BC}$ **d** $\overset{\frown}{AB}$



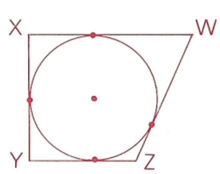
5 Can a parallelogram with a 100° angle be inscribed in a circle?

7 **a** If a rhombus is inscribed in a circle, what must be true about the rhombus?
b If a trapezoid is inscribed in a circle, what must be true about the trapezoid?

9 Given: $\angle B = 115°$, $\overset{\frown}{AD} = 60°$, $\overline{BC} \parallel \overline{EF}$
Find: **a** $\angle ADC$ **c** $\angle C$
b $\angle CDF$ **d** $\angle A$



11 Trapezoid WXYZ is circumscribed about circle O. $\angle X$ and $\angle Y$ are right $\angle$s, $XW = 16$, and $YZ = 7$. Find the perimeter of WXYZ.



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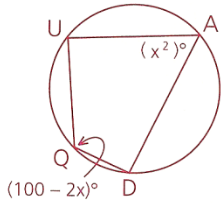
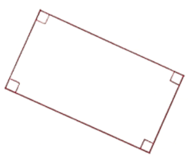
15 Given the figure shown, find m∠Q.

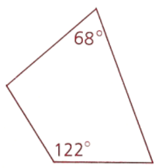


17 A quadrilateral can be inscribed in a circle only if a pair of opposite angles are supplementary. Which of the following quadrilaterals can be inscribed in a circle?

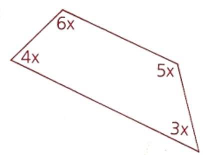
a



b



c

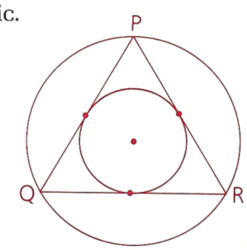


Challenge:

19 Equilateral triangle PQR is inscribed in one circle and circumscribed about another circle. The circles are concentric.

a If the radius of the smaller circle is 10, find the radius of the larger circle.

b In general, for an equilateral triangle, what is the ratio of the radius of the inscribed circle to the radius of the circumscribed circle?



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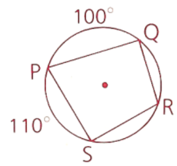
Classwork

Directions: Complete the 3 problems below. Each is worth 3 points. This classwork is worth 9 points.

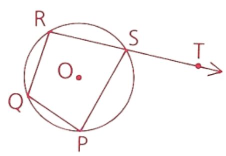
2 Given: $\widehat{PS} = 110°$, $\widehat{PQ} = 100°$

Find: m∠R and m∠P



4 Given: ⊙O

Prove: ∠Q ≅ ∠PST



6 Given: PQRST is a regular pentagon.
ABCDEF is a regular hexagon.

Find: **a** $m\widehat{PQ}$ **d** $m\widehat{BD}$

b $m\widehat{RT}$ **e** $m\widehat{DEA}$

c $m\widehat{AB}$