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Adv Geo - ______ 10-6: More Angle-Arc Theorems Date__________________

Objectives

After studying this section, you will be able to

- State the purposes of proof and provide examples that demonstrate each purpose
- Recognize congruent inscribed and tangent-chord angles
- Determine the measure of an angle inscribed in a semicircle
- Apply the relationship between the measures of a tangent-tangent angle and its minor arc

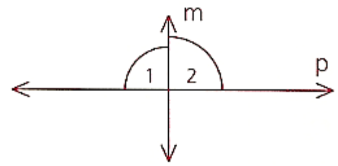
The Purposes of Proof Ch4

specific Two-column format General

Theorem 23 ***If two angles are both supplementary and congruent, then they are right angles.***

Given: $\angle 1 \cong \angle 2$

Prove: $\angle 1$ and $\angle 2$ are right angles.



Statements	Reasons
1. $\angle 1 \cong \angle 2$	1) Given
2. $m\angle 1 = m\angle 2$	2) $\cong\angle$s $\Rightarrow$ = measure (1)
3. $\angle 1$ supp $\angle 2$	3) st $\angle \Rightarrow$ supp $\angle$s
4. $m\angle 1 + m\angle 2 = 180°$	4) supp $\angle$s sum to 180° (3)
5. $m\angle 1 + m\angle 1 = 180°$	5) substitute (2, 4)
6. $2(m\angle 1) = 180°$	6) add (5)
7. $m\angle 1 = 90°$	7) divide (6)
8. $\angle 1$ is a right angle	8) If an angle measures 90°, then it is a right angle
9) $\angle 2$ rt $\angle$	9) substitute

Paragraph format

Proof: If two angles form a straight angle, then they are supplementary. Hence, $\angle 1$ is supplementary to $\angle 2$. Supplementary angles sum to 180°, thus $m\angle 1 + m\angle 2 = 180°$. It's given that $\angle 1 \cong \angle 2$. By substitution, $m\angle 1 + m\angle 1 = 180°$. By addition, $2(m\angle 1) = 180°$. By division, $m\angle 1 = 90°$. If an angle measures 90°, it is a right angle. Therefore $\angle 1$ is a right angle and so is $\angle 2$.

Now that we have proven the theorem, we can add it to our system and use it in the future. Now, whenever we come across two angles that form a straight angle, we can immediately state each is right angle (saving us steps in future proofs).

Verify the truth of a mathematical statement: The statements - specific to this problem - show logically, and step by step, why the final statement is true. (Note: Remember that we started the year with some logic: $a \Rightarrow b$ & $b \Rightarrow c$ $\therefore$ $a \Rightarrow c$.)

Explain why it is true: The reasons are generalizations that are already in our system. We know these to be always true.

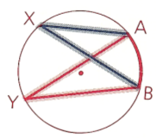
Communicate mathematical knowledge: The proof is not the format. The proof is the logical process of specifically showing why, step by step, and supporting each step with a trustworthy generalization.

Discover new mathematics: When we prove theorems, we get to add them to our system which makes it larger.

Create an axiomatic system: We started with a few assumptions. (For example, a point is a place in space. It has no length, width, or height.) We wrote definitions based on those assumptions, and used those definitions to prove theorems. Then we continued to use our existing axioms (that is definitions, theorems, and postulates) to prove more theorems. This process created an axiomatic system.

Theorem 89 ***If two inscribed or tangent-chord angles intercept the same arc, then they are congruent.***

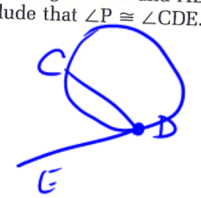
Given: X and Y are inscribed angles intercepting arc AB.
Conclusion: ∠X ≅ ∠Y

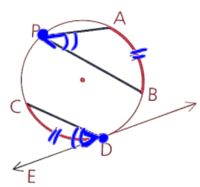


If an angle is inscribed then its measure is ½ of the measure of the arc it makes: $m\angle AXB = \frac{1}{2} m\widehat{AB}$ and $m\angle AYB = \frac{1}{2} m\widehat{AB}$. If two angles have the same measure then they are congruent. Therefore, ∠AXB ≅ ∠AYB. QED

Theorem 90 ***If two inscribed or tangent-chord angles intercept congruent arcs, then they are congruent.***

If $\overleftrightarrow{ED}$ is the tangent at D and $\widehat{AB} \cong \widehat{CD}$, we may conclude that ∠P ≅ ∠CDE.

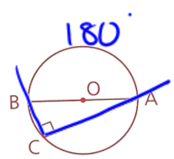




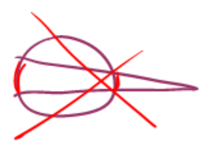
If an angle is inscribed then its measure is ½ of the measure of the arc it makes: $m\angle APB = \frac{1}{2} m\widehat{AB}$ and $\angle CDE = \frac{1}{2} m\widehat{CD}$. It's given that $\widehat{AB} \cong \widehat{CD}$. If two arcs are congruent then they have the same measure, so $m\widehat{AB} = m\widehat{CD}$. By substitution, both of these angles measure ½ m $\widehat{AB}$. If two angles have the same measure then they are congruent. Therefore, ∠APB ≅ ∠CDE. QED

Theorem 91 ***An angle inscribed in a semicircle is a right angle.***

Given: $\overline{AB}$ is a diameter of ⊙O.
Prove: ∠C is a right angle.

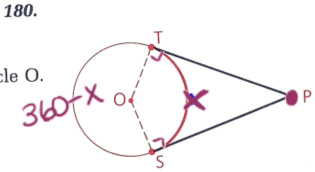


Diameter ⇒ ½ ⊙
½ ⊙ = 180°
180 = $\widehat{AB}$
$\frac{\widehat{AB}}{2} = \angle C$
∠C = 90°



Theorem 92 ***The sum of the measures of a tangent-tangent angle and its minor arc is 180.***

Given: $\overline{PT}$ and $\overline{PS}$ are tangent to circle O.
Prove: $m\angle P + m\widehat{TS} = 180$



Let $\widehat{TS} = x$. A circle measures 360°, so major arc $\widehat{TS} = (360 - x)°$.
Then by the formula, $m\angle P = \frac{(360-x)-x}{2}$.

Multiply: $2(m\angle P) = (360 - x) - x$.

Add: $2(m\angle P) = 360 - 2x$

Factor the right hand side: $2(m\angle P) = 2(180 - x)$

Divide: $m\angle P = 180 - x$

Add: $m\angle P \overset{+x}{=} 180$ ~~x~~

Substitute the symbols with words: (tan-tan angle) + (minor arc) = 180°

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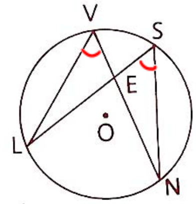
Examples

Problem 1 *Given:* ⊙O

Conclusion: $\triangle LVE \sim \triangle NSE$,
$EV \cdot EN = EL \cdot SE$

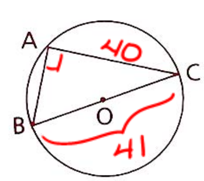


Proof

∠V makes $\overset{\frown}{LN}$
∠S makes $\overset{\frown}{LN}$
∠L forms $\overset{\frown}{VS}$
∠N forms $\overset{\frown}{VS}$

1 ⊙O	1 GIVEN
2 $\angle V \cong \angle S$	2 If 2 inscribed ∠s form same arc, then ≅∠s
3 $\angle L \cong \angle N$	3 same as 2
4 $\triangle LVE \sim \triangle NSE$	4 AA~
5 $\frac{EV}{SE} = \frac{EL}{EN}$	5 ~∠S ⇒ corr. sds. prop.
6 $EV \cdot EN = EL \cdot SE$	6 Means Extremes Product

Problem 2 *In circle O, $\overline{BC}$ is a diameter and the radius of the circle is* 20.5 mm. *Chord $\overline{AC}$ has a length of* 40 mm. *Find* AB.



Solution Book Since ∠A is inscribed in a semicircle, it is a right angle. By the Pythagorean Theorem,

The radius = 20.5, so the diameter BC = 41 mm.

If an inscribed angle forms a semicircle, then it is a right angle. Hence ∠ A is a right angle.

Label the diagram with appropriate marks and measures.

Pyth. Triple : 9, 40, 41 ∴ AB = 9

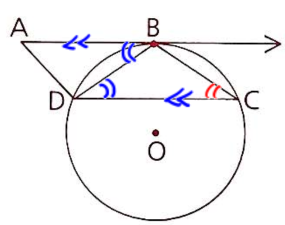
$AB^2 + 40^2 = 41^2$

$AB^2 = 1681 - 1600$

$AB = 9$

Problem 3 *Given:* ⊙O *with* $\overleftrightarrow{AB}$ *tangent at* B, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: $\angle C \cong \angle BDC$

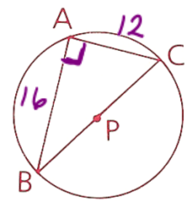


Proof

∠C forms $\overset{\frown}{DB}$
∠ABD forms $\overset{\frown}{DB}$

1 $\overleftrightarrow{AB}$ is tangent to ⊙O.	1 Given
2 $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$	2 Given
3 $\angle ABD \cong \angle BDC$	3 ∥ → ALT. INT ∠S ≅
4 $\angle C \cong \angle ABD$	4 If inscrib ∠s form same arc then ∠s ≅.
5 $\angle C \cong \angle BDC$	5 TRANS (3 & 4)

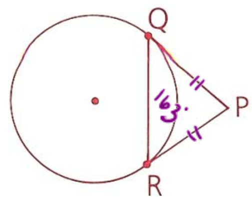
3 In $\odot P$, $\overline{BC}$ is a diameter, $AC = 12$ mm, and $BA = 16$ mm. Find the radius of the circle.



If an inscribed angle forms ½ circle, then the chord it creates is a diameter.
Given rt Δ with sides: (12, 16, BC).
Use the reduced triangle principle: 4 (3, 4, 5) such that BC = 20.
BC is the diameter. Then the radius is 10.

4 Given: $\overline{PQ}$ and $\overline{PR}$ are tangent segments.
$\overset{\frown}{QR} = 163°$

Find: **a** $\angle P$
b $\angle PQR$



(a) The tan-tan ∠ & the minor arc sum to 180°.

$\overset{\frown}{QR} + \angle P = 180°$
$\angle P = 180 - 163$
$\angle P = 17°$

(b) $\overline{QP} \cong \overline{PR}$ (2tan thm)

$\angle Q \cong \angle R$ ($\triangle \Rightarrow \triangle$)

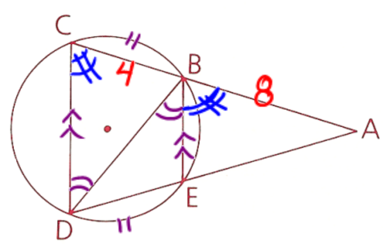
(Σ ∠s in Δ = 180°): $2x + 17 = 180$

Subtract: $2x = 163$

Divide: $x = 81\frac{1}{2} = \angle PQR$

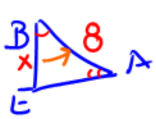
6 Given: $\overset{\frown}{BC} \cong \overset{\frown}{ED}$, $AB = 8$, $BC = 4$, $CD = 9$

a Are $\overline{BE}$ and $\overline{CD}$ parallel? yes
b Find BE. 6
c Is $\triangle ACD$ scalene? No



a) $\overset{\frown}{BC} \cong \overset{\frown}{ED}$ (given)
$\angle CDB \cong \angle DBE$ (≅ arcs ⇒ ≅ inscr ∠s)
$\overline{BE} \parallel \overline{CD}$ (alt int ∠s ≅ ⇒ ∥)

b) (∥ ⇒ corr ∠s ≅) $\angle C \cong \angle ABE$
$\angle A \cong \angle A$ (ref)
$\triangle ABE \sim \triangle ACD$ (AA~)



~Δs ⇒ corr sds prop

$\therefore \frac{9}{12} = \frac{x}{8}$, $\frac{3}{4} = \frac{x}{8}$, $x = 6$

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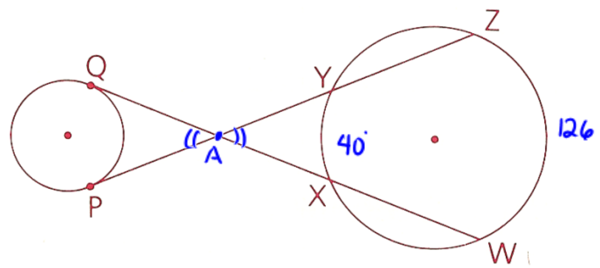
7 Given: $\overleftrightarrow{PY}$ and $\overleftrightarrow{QW}$ are tangents.

$\overset{\frown}{WZ} = 126°$, $\overset{\frown}{XY} = 40°$

Find: $\overset{\frown}{PQ}$



$m\angle YAX = \frac{126-40}{2} = \frac{86}{2} = 43°$

$\angle YAX \cong \angle QAP$ (vert $\angle$s $\Rightarrow \cong \angle$s)

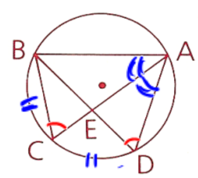
$\angle QAP + \overset{\frown}{QP} = 180°$

$43° + \overset{\frown}{QP} = 180°$

$\overset{\frown}{QP} = 137°$

18 Given: $\overset{\frown}{BC} \cong \overset{\frown}{CD}$

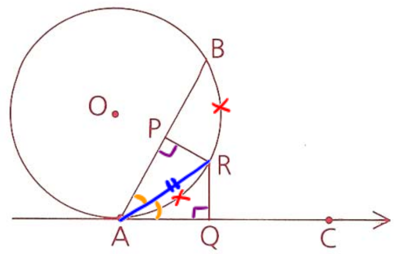
Conclusion: $\triangle ABC \sim \triangle AED$



1. $\overset{\frown}{BC} \cong \overset{\frown}{CD}$ — 1. Given
2. $\angle BAC \cong \angle CAD$ — 2. 2 $\cong$ arcs formed by inscribed $\angle$s $\Rightarrow \cong \angle$s
3. $\angle BCA \cong \angle BDA$ — 3. Same arc formed by inscrib $\angle$s $\Rightarrow \cong \angle$s
4. $\triangle ABC \sim \triangle AED$ — 4. AA $\sim$

19 Given: $\overleftrightarrow{AC}$ is tangent at A. $\angle APR$ and $\angle AQR$ are right $\angle$s. R is the midpoint of $\widehat{AB}$.

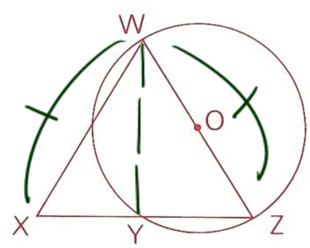
Conclusion: $\overline{PR} \cong \overline{RQ}$ (Hint: Draw $\overline{AR}$.)



1. $\overleftrightarrow{AC}$ tan @ A, $\angle APR$ & $\angle AQR$ rt$\angle$s	1. Given
2. $\angle APR \cong \angle AQR$	2. rt$\angle$s $\Rightarrow \cong \angle$s
3. Draw $\overline{AR}$	3. Aux
4. $\overline{AR} \cong \overline{AR}$	4. Ref
5. R mdpt $\widehat{AB}$	5. Given
6. $\widehat{BR} \cong \widehat{RA}$	6. mdpt $\Rightarrow \cong$ Arcs
7. $\angle BAR \cong \angle RAQ$	7. $\cong$ arcs $\Rightarrow$ 2 inscribed $\angle$s that form the arcs are $\cong$
8. $\triangle PAR \cong \triangle QAR$	8. AAS (2 7 4)
9. $\overline{PR} \cong \overline{RQ}$	9. CPCTC (8)

20 Given: $\triangle WXZ$ is isosceles, with $\overline{WX} \cong \overline{WZ}$. $\overline{WZ}$ is a diameter of $\odot O$.

Prove: Y is the midpoint of $\overline{XZ}$. (Hint: Draw $\overline{WY}$.)



1. $\triangle WXZ$ isos$\triangle$, $\overline{WX} \cong \overline{WZ}$ & $\overline{WZ}$ diameter $\odot O$.	1. Given
2. Draw $\overline{WY}$	2. Aux
3. $\angle WYZ$ rt$\angle$	3. Inscribed $\angle$ forms semi$\odot \Rightarrow$ rt$\angle$
4. $\overline{WY} \perp \overline{XZ}$	4. rt$\angle \Rightarrow \perp$
5. $\overline{WY}$ alt to $\overline{XZ}$	5. $\perp \Rightarrow$ alt
6. $\overline{WY}$ median to $\overline{XZ}$	6. in isos $\triangle$, the alt is also the median to the base
7. Y mdpt $\overline{XZ}$	7. median $\Rightarrow$ mdpt

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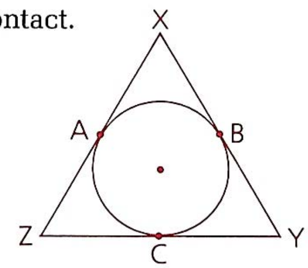
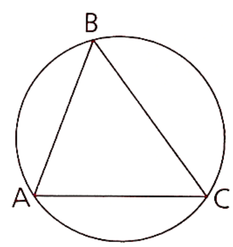
Homework

5 Given: A, B, and C are points of contact.
$\overset{\frown}{AB} = 145°$, $\angle Y = 48°$

Find: $\angle Z$



8 If $\triangle ABC$ is inscribed in a circle and $\overset{\frown}{AC} \cong \overset{\frown}{AB}$, tell whether each of the following must be true, could be true, or cannot be true.



a $\overline{AB} \cong \overline{AC}$

b $\overline{AC} \cong \overline{BC}$

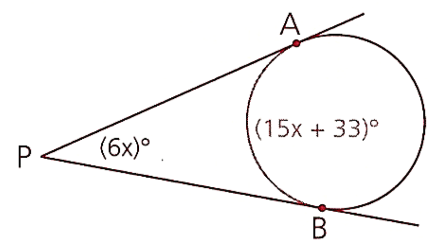
c $\overline{AB}$ and $\overline{AC}$ are equidistant from the center of the circle.

d $\angle B \cong \angle C$

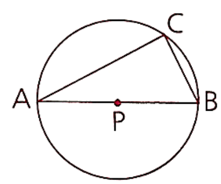
e $\angle BAC$ is a right angle.

f $\angle ABC$ is a right angle.

9 In the figure shown, find m∠P.



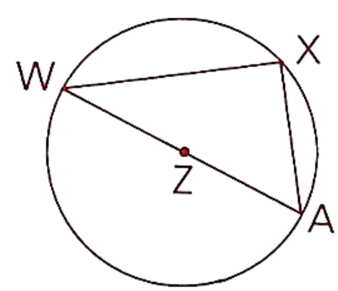
10 If $\overline{AB}$ is a diameter of ⊙P, CB = 1.5 m, and CA = 2 m, find the radius of ⊙P.



11 The radius of ⊙Z is 6 cm and $\overset{\frown}{WX}$ = 120°.

Find: **a** AX

b The perimeter of △WAX



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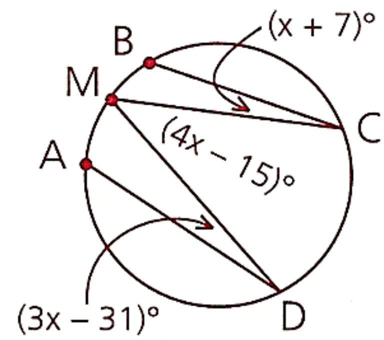
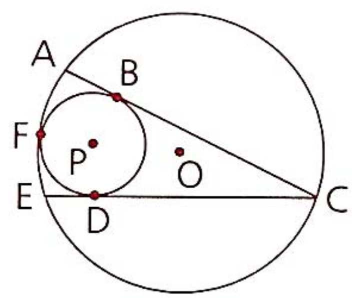
12 M is the midpoint of $\overset{\frown}{AB}$. Find m$\overset{\frown}{CD}$.



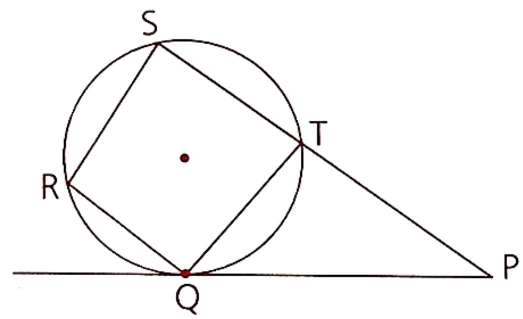
15 Quadrilateral ABCD is inscribed in circle O. AB = 12, BC = 16, CD = 10, and ∠ABC is a right angle. Find the measure of $\overline{AD}$ in simplified radical form.

16 Circles O and P are tangent at F. $\overline{AC}$ and $\overline{CE}$ are tangent to ⊙P at B and D. If $\overset{\frown}{DFB}$ = 223°, find $\overset{\frown}{AE}$.

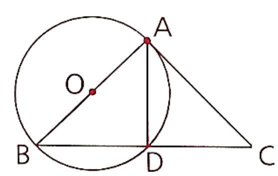


17 Given: $\angle S = 88°$, $\overset{\frown}{QT} = 104°$, $\overset{\frown}{ST} = 94°$, tangent $\overline{PQ}$

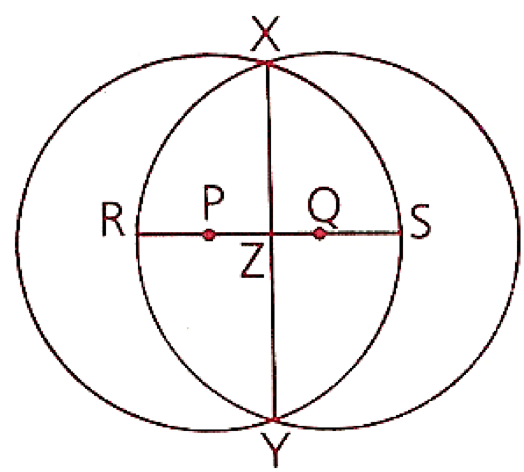
Find: **a** $\angle P$
b $\angle STQ$



21 Given: $\overline{AC}$ is tangent to $\odot O$ at A.
Conclusion: $\triangle ADC \sim \triangle BDA$



24 Given: $\odot P \cong \odot Q$,
$XY = 8$,
$RP = QS = 1$
Find: PQ



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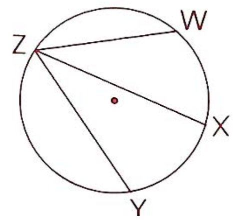
~~Classwork~~ Homework P 7 & 5 _____/12

Complete the problems on your own. Compare work with a partner. Discuss any differences, and revise. Hand in when completed (before the period ends).

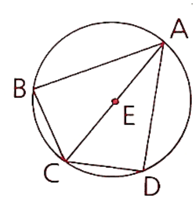
1 Given: X is the midpt. of $\widehat{WY}$.
Prove: $\overrightarrow{ZX}$ bisects $\angle WZY$.



(4 points, 1 pt per reason)

S	R
1. X is the midpoint of $\widehat{WY}$	1.
2. $\widehat{WX} \cong \widehat{XY}$	2.
3. $\angle WZX \cong \angle XZY$	3.
4. $\overrightarrow{ZX}$ bisects $\angle WZY$	4.

2 Given: $\odot E$ with diameter $\overline{AC}$, $\overline{BC} \cong \overline{CD}$
Conclusion: $\triangle ABC \cong \triangle ADC$



(5 points, holistic)

13 A rectangle with dimensions 18 by 24 is inscribed in a circle.
Find the radius of the circle.

(3 pts)