NAME Ms. Kresovic

Adv Geo - 2 10.5: Angles related to a circle

Objectives

After studying this section, you will be able to

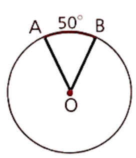
- Determine the measures of central angles
- Determine the measures of inscribed and tangent-chord angles
- Determine the measures of chord-chord angles
- Determine the measures of secant-secant, secant-tangent, and tangent-tangent angles

Angles with Vertices at the Center of a Circle

The measure of an angle whose sides intersect a circle is determined by the measure of its intercepted arcs. The location of the vertex of each angle is the key to remembering how to compute the measure of the angle.

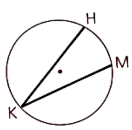
An angle with its vertex at the center of a circle is a central angle, already defined to be equal in measure to its intercepted arc (Section 10.3).

In ⊙O, $\widehat{AB} = 50°$, so $m\angle AOB = 50$.

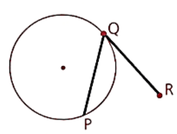


Angles with Vertices on a Circle

Two important types of angles whose vertices are on a circle are shown below.



$\angle HKM$ is an ***inscribed angle.***



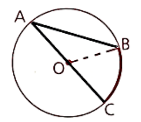
$\angle PQR$ is a ***tangent-chord angle.***

Definition An ***inscribed angle*** is an angle whose vertex is on a circle and whose sides are determined by two chords.

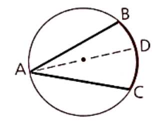
Definition A ***tangent-chord angle*** is an angle whose vertex is on a circle and whose sides are determined by a tangent and a chord that intersect at the tangent's point of contact.

Theorem 86 ***The measure of an inscribed angle or a tangent-chord angle (vertex on a circle) is one-half the measure of its intercepted arc.***

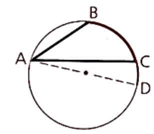
The proof of Theorem 86 for inscribed angles is unusual because three cases must be considered. Shown below are some key steps for each case in the proof that $m\angle BAC = \frac{1}{2}(m\widehat{BC})$.



Case 1:
The center lies on a side of the angle.
1 $m\angle BOC = m\widehat{BC}$
2 $\angle BOC = \angle BAC + \angle ABO$, so $m\angle BOC = 2(m\angle BAC)$



Case 2:
The center lies inside the angle.
1 Use case 1 twice.
2 Add ∠s and arcs.



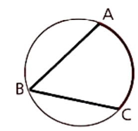
Case 3:
The center lies outside the angle.
1 Use case 1 twice.
2 Subtract ∠s and arcs.

Example 1 *Given:* $m\widehat{AC} = 112$
Find: $m\angle B$

$m\angle B = \frac{1}{2}(m\widehat{AC})$

$= \frac{1}{2} \cdot 112$

$= 56$

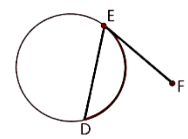


Example 2 *Given:* $\overline{FE}$ *is tangent at* E.
$m\widehat{DE} = 80$
Find: $m\angle DEF$

$m\angle DEF = \frac{1}{2}(m\widehat{DE})$

$= \frac{1}{2} \cdot 80$

$= 40$

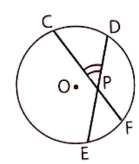


Angles with Vertices Inside, but Not at the Center of, a Circle

One type of angle other than a central angle has a vertex inside a circle.

Definition A ***chord-chord angle*** is an angle formed by two chords that intersect inside a circle but not at the center.

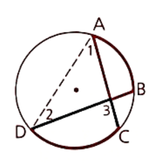
$\angle CPD$ is one of four chord-chord angles formed by chords $\overline{CF}$ and $\overline{DE}$ in circle O.



Theorem 87 ***The measure of a chord-chord angle is one-half the sum of the measures of the arcs intercepted by the chord-chord angle and its vertical angle.***

Notice that one-half the sum of the arc measures is the same as the *average* of the arc measures.

Given: ∠3 is a chord-chord angle.
Prove: $m\angle 3 = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$



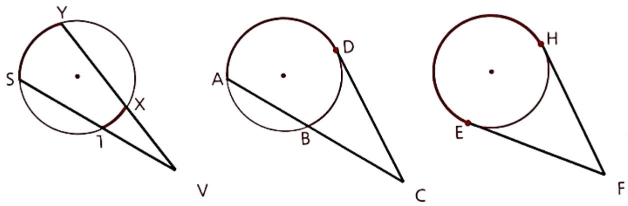
Here are two key steps in a proof of Theorem 87.

1 $m\angle 3 = m\angle 1 + m\angle 2$

2 $m\angle 3 = \frac{1}{2}(m\widehat{CD}) + \frac{1}{2}(m\widehat{AB})$

Angles with Vertices Outside a Circle

There are three types of angles having a vertex outside a circle and both sides intersecting the circle.



$\angle V$ is a *secant-secant angle.* $\angle C$ is a *secant-tangent angle.* $\angle F$ is a *tangent-tangent angle.*

Definition A ***secant-secant angle*** is an angle whose vertex is outside a circle and whose sides are determined by two secants.

Definition A ***secant-tangent angle*** is an angle whose vertex is outside a circle and whose sides are determined by a secant and a tangent.

Definition A ***tangent-tangent angle*** is an angle whose vertex is outside a circle and whose sides are determined by two tangents.

Theorem 88 ***The measure of a secant-secant angle, a secant-tangent angle, or a tangent-tangent angle (vertex outside a circle) is one-half the difference of the measures of the intercepted arcs.***

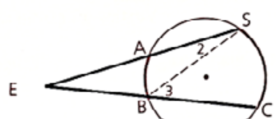
Key steps in a proof of Theorem 88 for secant-secant angles follow.

Prove: $m\angle E = \frac{1}{2}(m\widehat{SC} - m\widehat{AB})$

1 $m\angle 3 = m\angle E + m\angle 2$; solve for $m\angle E$.

2 $m\angle 2 = \frac{1}{2}(m\widehat{AB})$; $m\angle 3 = \frac{1}{2}(m\widehat{SC})$

3 Substitute and simplify.

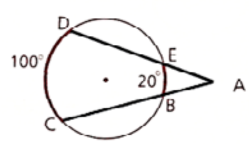


Example 1 Find $m\angle A$.

$m\angle A = \frac{1}{2}(m\widehat{CD} - m\widehat{BE})$

$= \frac{1}{2}(100 - 20)$

$= 40$



Example 2 Find $m\angle F$.

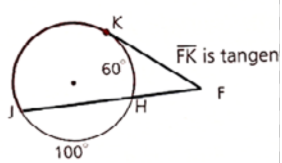
$m\widehat{JK} = 360 - 100 - 60$

$= 200$

$m\angle F = \frac{1}{2}(m\widehat{JK} - m\widehat{HK})$

$= \frac{1}{2}(200 - 60)$

$= 70$



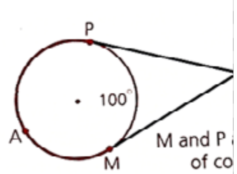
Example 3 Find $m\angle Q$.

$m\widehat{MAP} = 360 - 100 = 260$

$m\angle Q = \frac{1}{2}(m\widehat{MAP} - m\widehat{MP})$

$= \frac{1}{2}(260 - 100)$

$= 80$

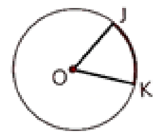


External → Empirical → Analytic

Book → patterns → Generalizations

Angle-Arc Summary

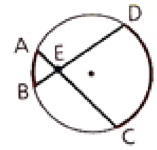
Central Angle



$m\angle KOJ = m\widehat{JK}$

Vertex at center ⟹ equal

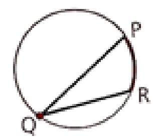
Chord-Chord Angle



$m\angle DEC = \frac{1}{2}(m\widehat{AB} + m\widehat{CD})$

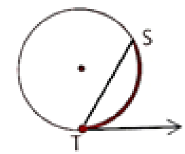
Vertex inside ⟹ half the sum

Inscribed Angle



$m\angle Q = \frac{1}{2}(m\widehat{PR})$

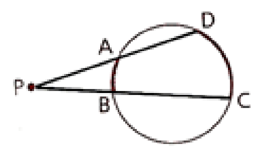
Tangent-Chord Angle



$m\angle T = \frac{1}{2}(m\widehat{ST})$

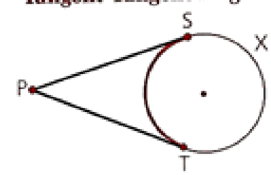
Vertex on circle ⟹ half the arc

Secant-Secant Angle



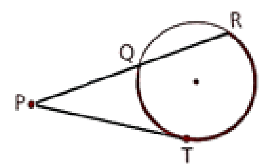
$m\angle P = \frac{1}{2}(m\widehat{CD} - m\widehat{AB})$

Tangent-Tangent Angle



$m\angle P = \frac{1}{2}(m\widehat{SXT} - m\widehat{ST})$

Secant-Tangent Angle



$m\angle P = \frac{1}{2}(m\widehat{RT} - m\widehat{QT})$

Vertex outside circle ⟹ half the difference

From the ASN:

If the vertex of the angle is ____ the circle	Then use this formula to find the angle's measure:
central	$\frown = \angle$
IN	$\angle = \frac{\frown + \frown}{2}$
ON	$\angle = \frac{\frown}{2}$
OUT	$\angle = \frac{\frown - \frown}{2}$

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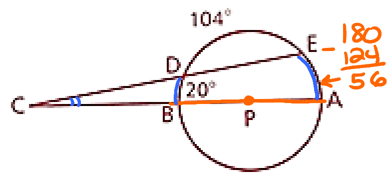
Class Examples

Problem 1 Given: $\overline{AB}$ is a diameter of ⊙P. $\overarc{BD} = 20°$, $\overarc{DE} = 104°$

Find: m∠C

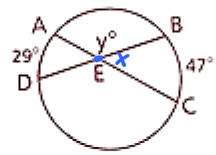


Solution First find m$\overarc{EA}$.

$m\angle C = \frac{56-20}{2} = \frac{36}{2} = 18°$

Problem 2 Find y.

Solution Find m∠BEC first.

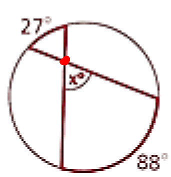


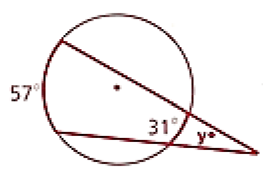
$m\angle BEC = \frac{\overarc{AD} + \overarc{BC}}{2} = \frac{29+47}{2}$

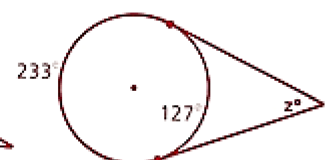
$= \frac{76}{2} = 35+3 = 38°$

stk: $x+y = 180°$ then $38+y = 180°$, $y = 142°$
-38

Problem 3 a Find x. b Find y. c Find z.







Solution

a $x = \frac{27+88}{2} = \frac{115}{2}$

$50+5+2.5$

$\boxed{57.5°}$

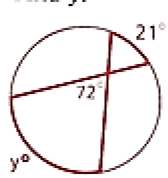
b $y = \frac{57-31}{2} = \frac{26}{2}$

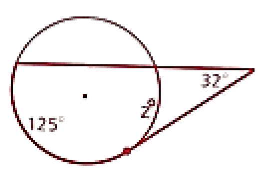
$\boxed{13°}$

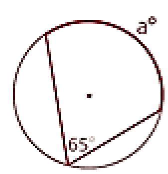
c $z = \frac{233-127}{2}$

$\frac{106}{2} = 53°$

Problem 4 a Find y. b Find z. c Find a.







Solution

a $72 = \frac{y+21}{2}$

$144 = y + 21$
$-21 \quad -21$

$123 = y$

b $32 = \frac{125-z}{2}$

$64 = 125 - z$
$+z \; -64 \quad -64 \quad +z$

$z = 61$

c $\angle = \frac{\frown}{2}$

$65 = \frac{a}{2}$

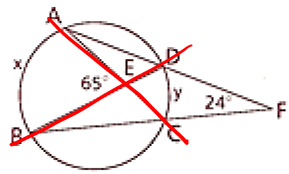
$130° = a$

Problem 5 *Find* $m\widehat{AB}$ *and* $m\widehat{CD}$.

Solution

$65 = \frac{x+y}{2}$; $x+y = 130'$

$24 = \frac{x-y}{2}$; $x - y = 48$



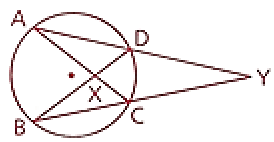
$$\begin{cases} x + y = 130 \\ x - y = 48 \end{cases}$$

$2x = 178$

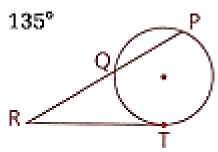
$x = 89$ then $89 + y = 130$ $m\widehat{AB} = 89°$

$y = 41$ $\therefore m\widehat{DC} = 41°$

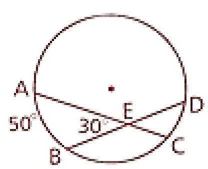
7 Given: $\widehat{AB} = 108°$, $\widehat{CD} = 62°$
Find: $\angle AXB$ and $\angle Y$



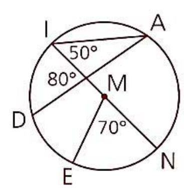
8 Given: $\widehat{TP} = 170°$, $\widehat{PQ} = 135°$
Find: $\angle R$



9 Given: $\angle AEB = 30°$,
$\widehat{AB} = 50°$
Find: $\widehat{CD}$



17 If a point is chosen at random on $\odot M$, what is the probability that it lies on
a $\widehat{IAN}$ **b** $\widehat{AN}$ **c** $\widehat{ID}$ **d** $\widehat{IE}$



Problem Set B

18 Given: $\overleftrightarrow{VQ}$ is tangent to $\odot O$ at Q.
$\overline{QS}$ is a diameter of $\odot O$.
$\widehat{PQ} = 115°$; $\angle RPS = 36°$

Find: **a** $\angle R$ **e** $\angle QPR$ **i** $\widehat{PRQ}$
b $\angle S$ **f** $\angle QPS$ **j** $\widehat{RSP}$
c $\widehat{SR}$ **g** $\angle QTP$ **k** $\angle VQS$
d $\widehat{QR}$ **h** $\angle PQV$ **l** $\angle QOP$