Name STU DENT Ms. Kresovic

Adv Geo – period 5 Date 3/17

10.4 Secants and Tangents

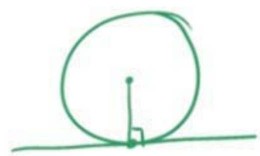
Objectives

After studying this section, you will be able to

- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents

Definition A ***secant*** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.) SEC

Definition A ***tangent*** is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact***.

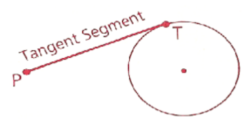


Postulate ***A tangent line is perpendicular to the radius drawn to the point of contact.*** RAD $\cap$ TAN $\Rightarrow \perp$

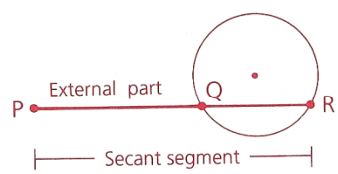
intersects

Postulate ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.*** $\perp \Rightarrow$ RAD $\cap$ TAN

Definition A ***tangent segment*** is the part of a tangent line between the point of contact and a point outside the circle.



Definition A ***secant segment*** is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

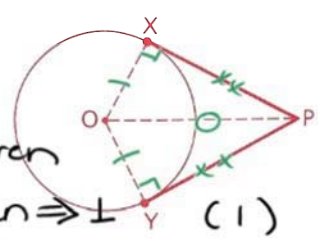


Definition The ***external part*** of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

Theorem 85 ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)***

Given: $\odot O$;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

Prove: $\overline{PX} \cong \overline{PY}$



1. $\odot O$, $\overline{PX}$ & $\overline{PY}$ tan $\odot O$ 1. given
2. $\overline{PX} \perp \overline{OX}$ & $\overline{PY} \perp \overline{OY}$ 2. tan $\Rightarrow \perp$ (1)

RTL 3. $\angle OXP$ & $\angle OYP$ rt$\angle$s 3. $\perp \Rightarrow$ RT$\angle$S (2)

H 4. $\overline{OP} \cong \overline{OP}$ 4. REF

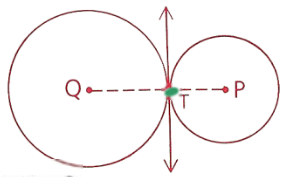
L 5. $\overline{OX} \cong \overline{OY}$ 5. $\odot \Rightarrow \cong$ RADII (1)

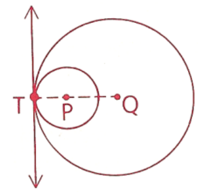
6. $\triangle OXP \cong \triangle OYP$ 6. HL (3,4,5)
7. $\overline{PX} \cong \overline{PY}$ 7. CPCTC (6)

THIS IS THE PROOF OF THM 85. NOW WE CAN USE IT: 2-TAN $\Rightarrow \cong$ SEGS

Tangent Circles

Definition ***Tangent circles*** are circles that intersect each other at exactly one point.

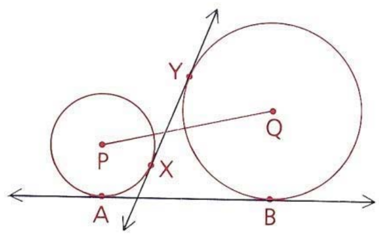




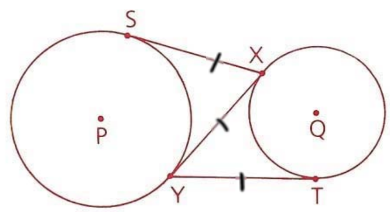
TODAY'S FOCUS

Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers.
$\overleftrightarrow{XY}$ is a ***common internal tangent.***
$\overleftrightarrow{AB}$ is a ***common external tangent.***



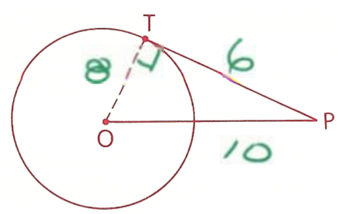
Problem 1 *Given:* $\overline{XY}$ *is a common internal tangent to* ⓢ P *and* Q *at* X *and* Y.
$\overline{XS}$ *is tangent to* ⊙P *at* S.
$\overline{YT}$ *is tangent to* ⊙Q *at* T.
Conclusion: $\overline{XS} \cong \overline{YT}$



Proof

1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q.	1 Given
2 $\overline{XY}$ is tangent to ⓢ P and Q.	2 Given
3 $\overline{XS} \cong \overline{XY}$	3 2 TAN ⇒ ≅ SEGS
4 $\overline{XY} \cong \overline{YT}$	4 2 TAN ⇒ ≅ SEGS
5 $\overline{XS} \cong \overline{YT}$	5 TRANSITIVE (3&4)

Problem 2 $\overleftrightarrow{TP}$ *is tangent to circle* O *at* T.
The radius of circle O *is* 8 mm.
Tangent segment $\overline{TP}$ *is* 6 mm *long.*
Find the length of $\overline{OP}$.



Solution

6, 8, ___

Common-Tangent Procedure

1 Draw the segment joining the centers.

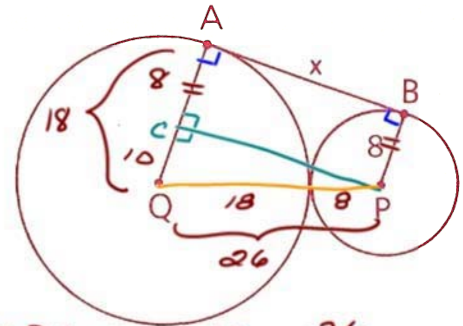
2 Draw the radii to the points of contact.

3 Through the center of the smaller circle, draw a line parallel to the common tangent.

4 Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle.

5 Use the Pythagorean Theorem and properties of a rectangle.

Problem 3 *A circle with a radius of* 8 cm *is externally tangent to a circle with a radius of* 18 cm. *Find the length of a common external tangent.*



Solution

① rad ∩ tan ⇒ ⊥

② Draw Rect, that is DRAW CP ∥ AB

③ Draw Hypotenuse

△QCP: x 10 26

2(12 5 13)

∴ x = 24cm

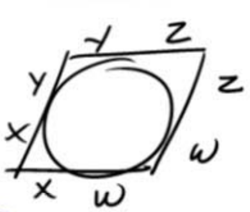
SAVE FOR TOMORROW

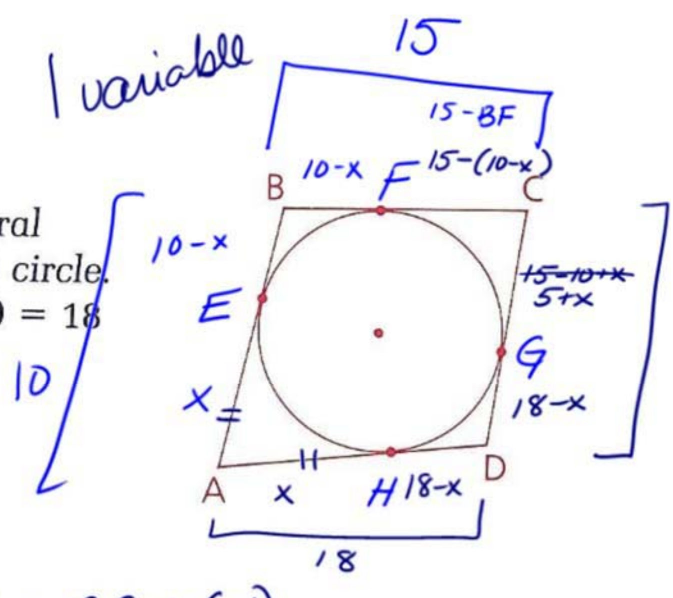
1 variable

Problem 4 A walk-around problem:

Given: Each side of quadrilateral ABCD is tangent to the circle. AB = 10, BC = 15, AD = 18

Find: CD





System

G: $\begin{cases} x + y = 10 \\ y + z = 15 \\ x + w = 18 \end{cases}$ F: w + z

y = 10 − x

then 10 − x + z = 15

z = x + 5

x + y = 10 → x = 10 − y

10 − y + w = 18

w = 8 + y

then w + z = 8 + y + x + 5 → x + y + 13 = 10 + 13 = 23

CD = CG + GD

= 5 + x + 18 − x

= 5 + 18

= 23

AMDG

Name ________________________

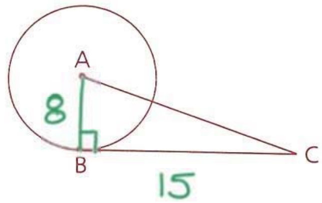
Ms. Kresovic

Adv Geo – period _____

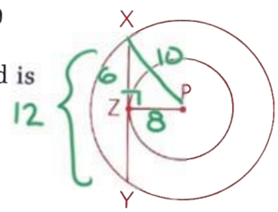
Date ___________________

Homework 10.4 Secants and Tangents

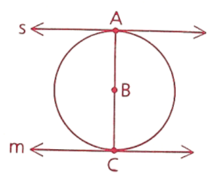
1 The radius of ⊙A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$. $= 17$



2 Concentric circles with radii 8 and 10 have center P.
$\overline{XY}$ is a tangent to the inner circle and is a chord of the outer circle.
Find $\overline{XY}$. (Hint: Draw $\overline{PX}$ and $\overline{PY}$.)



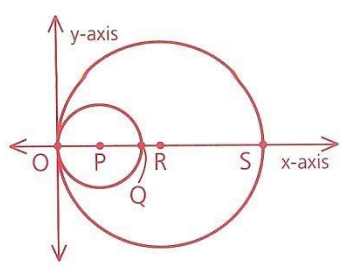
4 Given: $\overline{AC}$ is a diameter of ⊙B.
Lines s and m are tangents to the ⊙ at A and C.
Conclusion: s ∥ m



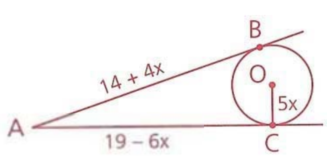
1. $\overline{AC}$ dia ⊙B, s & m tan ⊙ @ A & C | 1. given
2. $\overline{AB} \perp s$ & $\overline{BC} \perp m$ | 2. tan ⇒ ⊥
3. s ∥ m | 3. If 2 lines ⊥ to 3rd then 2 lines ∥.

5 ⊙P and ⊙R are internally tangent at O.
P is at (8, 0) and R is at (19, 0).
a Find the coordinates of Q and S. (38,0) (16,0)
b Find the length of $\overline{QR}$.

$19 - 16 = \boxed{3}$



6 $\overline{AB}$ and $\overline{AC}$ are tangents to ⊙O, and OC = 5x. Find OC.



2 tan ⇒ ≅ seg

$AB = AC$

$14 + 4x = 19 - 6x$

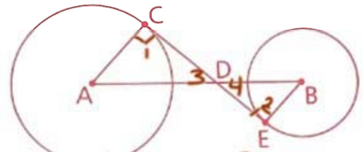
$10x = 5$

$x = 1/2$ such that $OC = \frac{5}{2}$

7 Given: $\overline{CE}$ is a common internal tangent to circles A and B at C and E.

Prove: **a** $\angle A \cong \angle B$

b $\frac{AD}{BD} = \frac{CD}{DE}$

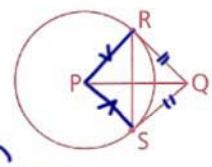


1. $\overline{CE}$ common int tan ⊙s A & B @ C & E — 1. given
2. $\overline{AC}$ & $\overline{BE} \perp \overline{CE}$ — 2. tan ⇒ ⊥
3. ∠1 & ∠2 rt∠s — 3. ⊥ ⇒ rt∠s
4. ∠1 ≅ ∠2 — 4. rt∠s ⇒ ≅∠s
5. ∠3 ≅ ∠4 — 5. vert∠s ⇒ ≅∠s
6. △CDA ~ △EDB — 6. AA~ (4,5)
7. ∠A ≅ ∠B — 7. ~△s ⇒ corr ∠s ≅

8. $\frac{AD}{BD} = \frac{CD}{DE}$ — 8. ~△s ⇒ corr sds prop.

8 Given: $\overline{QR}$ and $\overline{QS}$ are tangent to ⊙P at points R and S.

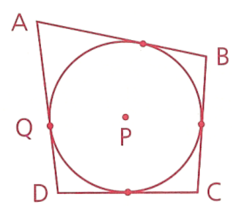
Prove: $\overline{PQ} \perp \overline{RS}$ (Hint: This can be proved in just a few steps.)



1. $\overline{QR}$ & $\overline{QS}$ tan ⊙P @ R & S — 1. Given
2. $\overline{QR} \cong \overline{QS}$ — 2. 2tan ⇒ ≅ segs
3. Draw $\overline{PR}$ & $\overline{PS}$ — 3. aux
4. $\overline{PR} \cong \overline{PS}$ — 4. ⊙ ⇒ radii ≅
5. $\overline{PQ} \perp \overline{RS}$ — 5. = dist ⇒ ⊥ bis (from ch4)

Skip, Save for Friday

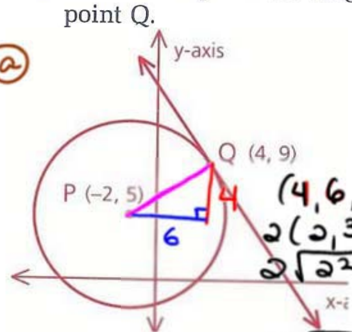
10 ⊙P is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.



11 **a** Find the radius of ⊙P. ← length

b Find the slope of the tangent to ⊙P at point Q.

(a)



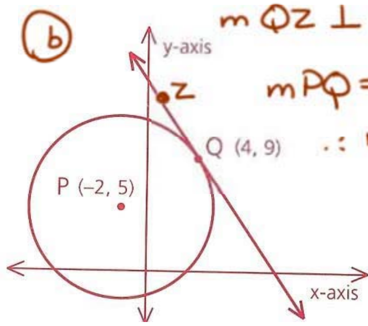
(4, 6, ?)

2(2, 3, —)

$2\sqrt{2^2+3^2}$

$2\sqrt{13}$

(b)



m QZ ⊥ m PQ

$mPQ = \frac{9-5}{4+2} = \frac{4}{6} = \frac{2}{3}$

$\therefore m QZ = \boxed{-\frac{3}{2}}$

12 Two concentric circles have radii 3 and 7. Find, to the nearest hundredth, the length of a chord of the larger circle that is tangent to the smaller circle. (See problem 2 for a diagram.)

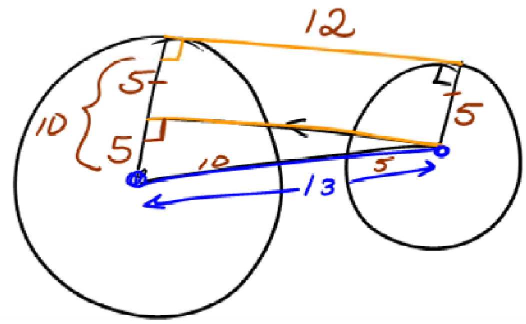
Thursday's homework Stops HERE

13 The centers of two circles of radii 10 cm and 5 cm are 13 cm apart.

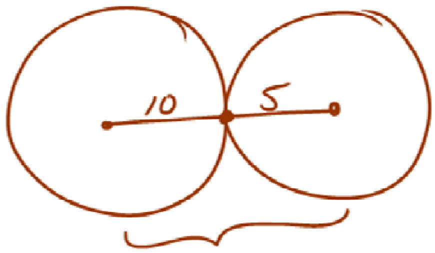
a Find the length of a common external tangent. (Hint: Use the common-tangent procedure.) 12

b Do the circles intersect?

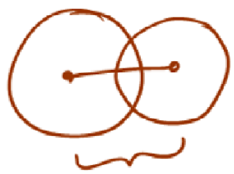
Yes, twice



$d > 15 \Rightarrow$ no $\cap$



$15 \Rightarrow$ ⊙s tan

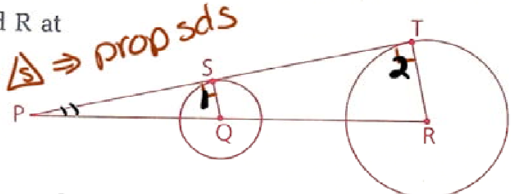


$13 < 15 \Rightarrow 2 \cap$s

14 The centers of two circles with radii 3 and 5 are 10 units apart. Find the length of a common internal tangent. (Hint: Use the common-tangent procedure.)

15 Given: $\overline{PT}$ is tangent to ⓢ Q and R at points S and T.

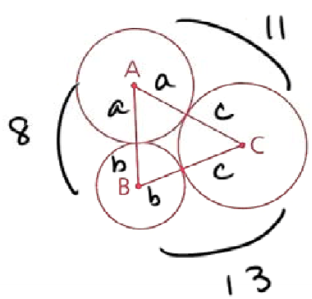
Conclusion: $\frac{PQ}{PR} = \frac{SQ}{TR}$ ← ~△s ⇒ prop sds



1. $\overline{PT}$ tan ⓢ Q & R @ S & T	1. Given
2. $\overline{SQ} \perp \overline{PT}$ & $\overline{TR} \perp \overline{PT}$	2. tan ⇒ ⊥
3. ∠1 & ∠2 rt∠s	3. ⊥ ⇒ rt∠s
A 4. ∠1 ≅ ∠2	4. rt∠s ⇒ ≅∠s
A 5. ∠P ≅ ∠P	5. refl
6. △PSQ ~ △PTR	6. AA ~ (4 5)
7. PQ/PR = SQ/TR	7. ~△s ⇒ corr sds prop.

16 Given: Tangent ⓢ A, B, and C, AB = 8, BC = 13, AC = 11

Find: The radii of the three ⓢ (Hint: This is a walk-around problem.)



$2a + 2b + 2c = 8 + 13 + 11$

$2(a + b + c) = 32$

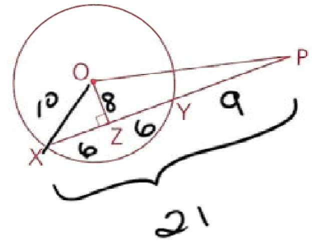
$a + b + c = 16$

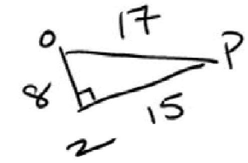
$a + b = 8$	$b + c = 13$	$a + c = 11$
& $a + b + c = 16$	& $a + b + c = 16$	& $a + b + c = 16$
$8 + c = 16$	$a + 13 = 16$	$11 + b = 16$
$c = 8$	$a = 3$	$b = 5$

17 The radius of ⊙O is 10.
The secant segment $\overline{PX}$ measures 21 and is 8 units from the center of the ⊙.

a Find the external part (PY) of the secant segment. 9

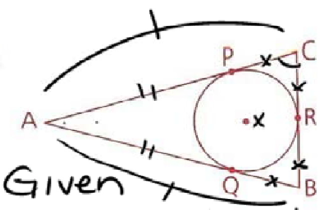
b Find OP. 17





$\overline{AP}$ & $\overline{AQ}$ tan ⊙X

18 Given: △ABC is isosceles, with base $\overline{BC}$.
Conclusion: $\overline{BR} \cong \overline{RC}$



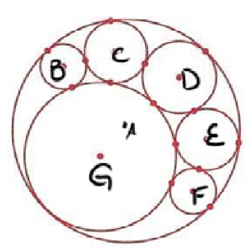
1. △ABC isos, $\overline{AP}$ & $\overline{AQ}$ tan ⊙X — 1. Given
2. $\overline{AC} \cong \overline{AB}$ — 2. isos ⇒ ≅ sds
3. $\overline{AP} \cong \overline{AQ}$ — 3. If you draw 2 tan from ext pt then they are ≅ (2 tan thm)
4. $\overline{PC} \cong \overline{QB}$ — 4. Subtract
5. $\overline{PC} \cong \overline{CR}$ & $\overline{QB} \cong \overline{BR}$ — 5. same as 3
6. $\overline{CR} \cong \overline{BR}$ — 6. trans (4 & 5)

19 If two of the seven circles are chosen at random, what is the probability that the chosen pair are

a Internally tangent?

b Externally tangent?

c Not tangent?



Let Big ⊙ ⇒ ⊙A

AB AC AD AE AF AG
BC BD BE BF BG
CD CE CF CG
DE DF DG
EF EG
FG

21 TOTAL

a) AB AC AD AE AF AG
BC BD BE BF BG
CD CE CF CG
DE DF DG
EF EG
FG

6/21 = 2/7

b) AB AC AD AE AF AG
BC BD BE BF BG
CD CE CF CG
DE DF DG
EF EG
FG

9/21 = 3/7

c) AB AC AD AE AF AG
BC BD BE BF BG
CD CE CF CG
DE DF DG
EF EG
FG

$\frac{6}{21} = \frac{2}{7}$