AMDG

Name STU DENT  Ms. Kresovic

Adv Geo – period 5  Date 3/17

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10.4 Secants and Tangents

## Objectives

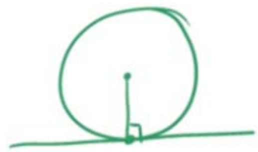
*After studying this section, you will be able to*

- Identify secant and tangent lines
- Identify secant and tangent segments
- Distinguish between two types of tangent circles
- Recognize common internal and common external tangents

**Definition** A ***secant*** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)

SEC

**Definition** A ***tangent*** is a line that intersects a circle at exactly one point. This point is called the ***point of tangency*** or ***point of contact.***



**Postulate** ***A tangent line is perpendicular to the radius drawn to the point of contact.***

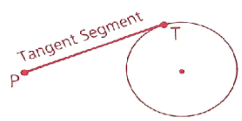
RAD ∩ TAN ⇒ ⊥

intersects

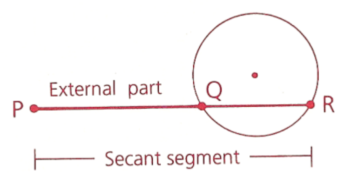
**Postulate** ***If a line is perpendicular to a radius at its outer endpoint, then it is tangent to the circle.***

⊥ ⇒ RAD ∩ TAN

**Definition** A ***tangent segment*** is the part of a tangent line between the point of contact and a point outside the circle.




**Definition** A ***secant segment*** is the part of a secant line that joins a point outside the circle to the farther intersection point of the secant and the circle.

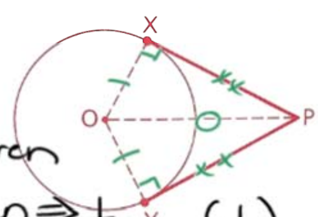



**Definition** The ***external part*** of a secant segment is the part of a secant line that joins the outside point to the nearer intersection point.

**Theorem 85** ***If two tangent segments are drawn to a circle from an external point, then those segments are congruent. (Two-Tangent Theorem)***

Given: ⊙O;
$\overline{PX}$ and $\overline{PY}$ are tangent segments.

Prove: $\overline{PX} \cong \overline{PY}$



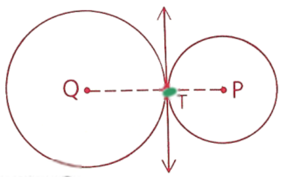

| | | |
|---|---|---|
| | 1. ⊙O, $\overline{PX}$ & $\overline{PY}$ tan ⊙O | 1. given |
| | 2. $\overline{PX} \perp \overline{OX}$ & $\overline{PY} \perp \overline{OY}$ | 2. tan ⇒ ⊥ (1) |
| RT∠ | 3. ∠OXP & ∠OYP rt∠s | 3. ⊥ ⇒ RT∠s (2) |
| H | 4. $\overline{OP} \cong \overline{OP}$ | 4. REF |
| L | 5. $\overline{OX} \cong \overline{OY}$ | 5. ⊙ ⇒ ≅ RADII (1) |
| | 6. △OXP ≅ △OYP | 6. HL (3,4,5) |
| | 7. $\overline{PX} \cong \overline{PY}$ | 7. CPCTC (6) |

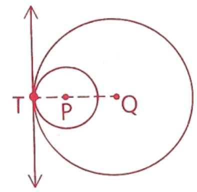
THIS IS THE PROOF OF THM 85. NOW WE CAN USE IT:

2-TAN ⇒ ≅ SEGS

## Tangent Circles

**Definition** *Tangent circles* are circles that intersect each other at exactly one point.

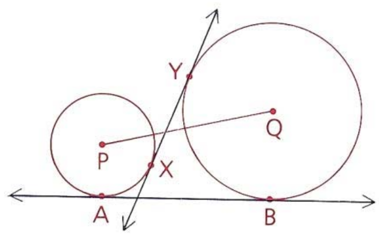


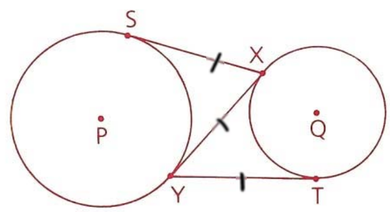

TODAY'S FOCUS

## Common Tangents

$\overleftrightarrow{PQ}$ is the line of centers.
$\overleftrightarrow{XY}$ is a ***common internal tangent.***
$\overleftrightarrow{AB}$ is a ***common external tangent.***



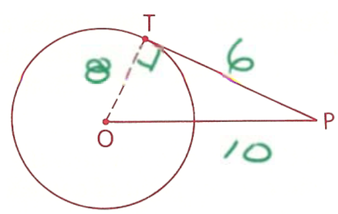

**Problem 1** *Given:* $\overline{XY}$ *is a common internal tangent to ⓢ P and Q at X and Y.*
$\overline{XS}$ *is tangent to ⊙P at S.*
$\overline{YT}$ *is tangent to ⊙Q at T.*
*Conclusion:* $\overline{XS} \cong \overline{YT}$




**Proof**

| | |
|---|---|
| 1 $\overline{XS}$ is tangent to ⊙P. $\overline{YT}$ is tangent to ⊙Q. | 1 Given |
| 2 $\overline{XY}$ is tangent to ⓢ P and Q. | 2 Given |
| 3 $\overline{XS} \cong \overline{XY}$ | 3 2 TAN ⇒ ≅ SEGS |
| 4 $\overline{XY} \cong \overline{YT}$ | 4 2 TAN ⇒ ≅ SEGS |
| 5 $\overline{XS} \cong \overline{YT}$ | 5 TRANSITIVE (3&4) |

**Problem 2** $\overleftrightarrow{TP}$ *is tangent to circle O at T.*
*The radius of circle O is 8 mm.*
*Tangent segment* $\overline{TP}$ *is 6 mm long.*
*Find the length of* $\overline{OP}$.

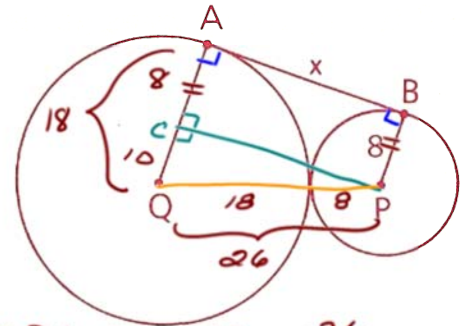



**Solution**

6, 8, ——

| Common-Tangent Procedure |
| --- |
| 1 Draw the segment joining the centers. |
| 2 Draw the radii to the points of contact. |
| 3 Through the center of the smaller circle, draw a line parallel to the common tangent. |
| 4 Observe that this line will intersect the radius of the larger circle (extended if necessary) to form a rectangle and a right triangle. |
| 5 Use the Pythagorean Theorem and properties of a rectangle. |

**Problem 3** *A circle with a radius of 8 cm is externally tangent to a circle with a radius of 18 cm. Find the length of a common external tangent.*




**Solution**

① rad ∩ tan ⇒ ⊥

② Draw Rect, that is DRAW CP || AB

③ Draw Hypotenuse

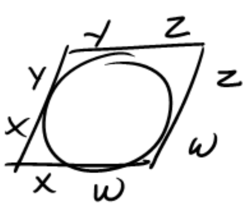
ΔQCP: x 10 26

2( 12 5 13 )

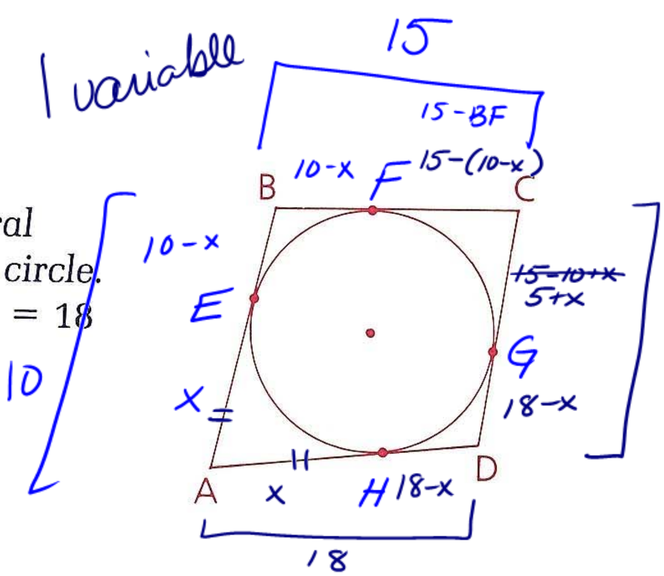
∴ x = 24cm

SAVE FOR TOMORROW

**Problem 4** A walk-around problem:
Given: Each side of quadrilateral ABCD is tangent to the circle. AB = 10, BC = 15, AD = 18
Find: CD




1 variable




System

G: $\begin{cases} x + y = 10 \\ y + z = 15 \\ x + w = 18 \end{cases}$ F: w + z

y = 10 − x

then 10 − x + z = 15

z = x + 5

x + y = 10 → x = 10 − y

10 − y + w = 18

w = 8 + y

then w + z = 8 + y + x + 5 → x + y + 13 = 10 + 13 = 23

CD = CG + GD

= 5 + x + 18 − x

= 5 + 18

= 23

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Name ________________________

Ms. Kresovic

Adv Geo – period _____

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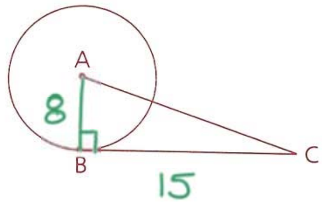
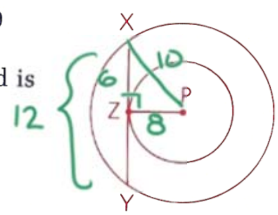
Homework 10.4 Secants and Tangents

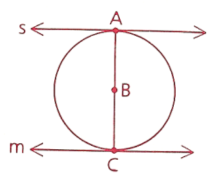
**1** The radius of ⊙A is 8 cm.
Tangent segment $\overline{BC}$ is 15 cm long.
Find the length of $\overline{AC}$. $= 17$




**2** Concentric circles with radii 8 and 10 have center P.
$\overline{XY}$ is a tangent to the inner circle and is a chord of the outer circle.
Find $\overline{XY}$. (Hint: Draw $\overline{PX}$ and $\overline{PY}$.)



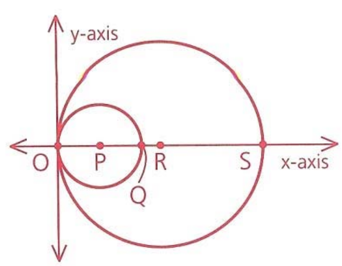

**4** Given: $\overline{AC}$ is a diameter of ⊙B.
Lines s and m are tangents to the ⊙ at A and C.
Conclusion: s ∥ m



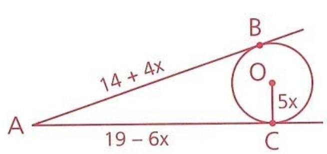

1. $\overline{AC}$ dia ⊙B, s & m tan ⊙ @ A & C | 1. given
2. $\overline{AB} \perp s$ & $\overline{BC} \perp m$ | 2. tan ⇒ ⊥
3. s ∥ m | 3. If 2 lines ⊥ to 3rd then 2 lines ∥.

**5** ⊙P and ⊙R are internally tangent at O.
P is at (8, 0) and R is at (19, 0).
**a** Find the coordinates of Q and S. (38,0) (16,0)
**b** Find the length of $\overline{QR}$.

$19 - 16 = \boxed{3}$




**6** $\overline{AB}$ and $\overline{AC}$ are tangents to ⊙O, and OC = 5x. Find OC.




2 tan ⇒ ≅ seg

$AB = AC$

$14 + 4x = 19 - 6x$

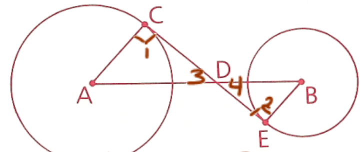
$10x = 5$

$x = \frac{1}{2}$ such that $OC = \frac{5}{2}$

**7** Given: $\overline{CE}$ is a common internal tangent to circles A and B at C and E.

Prove: **a** $\angle A \cong \angle B$

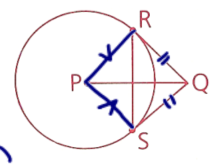
**b** $\frac{AD}{BD} = \frac{CD}{DE}$




| | |
|---|---|
| 1. $\overline{CE}$ common int tan ⓢ A & B @ C & E | 1. given |
| 2. $\overline{AC}$ & $\overline{BE} \perp \overline{CE}$ | 2. tan ⇒ ⊥ |
| 3. ∠1 & ∠2 rt∠s | 3. ⊥ ⇒ rt∠s |
| 4. ∠1 ≅ ∠2 | 4. rt∠s ⇒ ≅∠s |
| 5. ∠3 ≅ ∠4 | 5. vert∠s ⇒ ≅∠s |
| 6. △CDA ~ △EDB | 6. AA~ (4,5) |
| 7. ∠A ≅ ∠B | 7. ~ⓢ ⇒ corr∠s ≅ |
| 8. $\frac{AD}{BD} = \frac{CD}{DE}$ | 8. ~ⓢ ⇒ corr sds prop. |

**8** Given: $\overline{QR}$ and $\overline{QS}$ are tangent to ⊙P at points R and S.

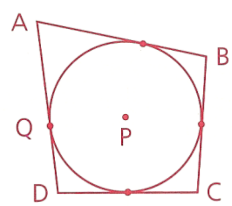
Prove: $\overline{PQ} \perp \overline{RS}$ (Hint: This can be proved in just a few steps.)




| | |
|---|---|
| 1. $\overline{QR}$ & $\overline{QS}$ tan ⊙P @ R & S | 1. Given |
| 2. $\overline{QR} \cong \overline{QS}$ | 2. 2tan ⇒ ≅ segs |
| 3. Draw $\overline{PR}$ & $\overline{PS}$ | 3. aux |
| 4. $\overline{PR} \cong \overline{PS}$ | 4. ⊙ ⇒ radii ≅ |
| 5. $\overline{PQ} \perp \overline{RS}$ | 5. = dist ⇒ ⊥ bis (from ch4) |

Skip, Save for Friday

**10** ⊙P is tangent to each side of ABCD. AB = 20, BC = 11, and DC = 14. Let AQ = x and find AD.

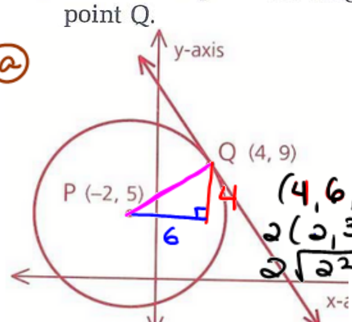



1

**11** **a** Find the radius of ⊙P. ← length

**b** Find the slope of the tangent to ⊙P at point Q.

ⓐ



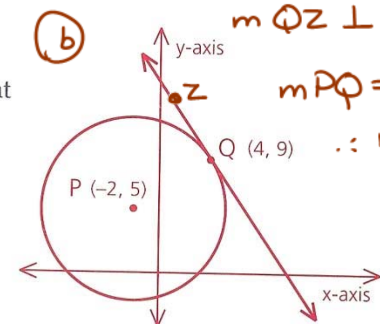

(4, 6, ?)

2(2, 3, —)

$2\sqrt{2^2+3^2}$

$2\sqrt{13}$

ⓑ




$mQZ \perp mPQ$

$mPQ = \frac{9-5}{4+2} = \frac{4}{6} = \frac{2}{3}$

$\therefore mQZ = \boxed{-\frac{3}{2}}$

**12** Two concentric circles have radii 3 and 7. Find, to the nearest hundredth, the length of a chord of the larger circle that is tangent to the smaller circle. (See problem 2 for a diagram.)

Thursday's homework Stops HERE

**13** The centers of two circles of radii 10 cm and 5 cm are 13 cm apart.

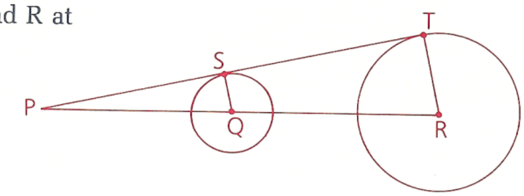
**a** Find the length of a common external tangent. (Hint: Use the common-tangent procedure.)

**b** Do the circles intersect?

**14** The centers of two circles with radii 3 and 5 are 10 units apart. Find the length of a common internal tangent. (Hint: Use the common-tangent procedure.)

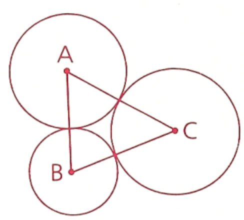
**15** Given: $\overline{PT}$ is tangent to ⓢ Q and R at points S and T.

Conclusion: $\frac{PQ}{PR} = \frac{SQ}{TR}$




**16** Given: Tangent ⓢ A, B, and C, AB = 8, BC = 13, AC = 11

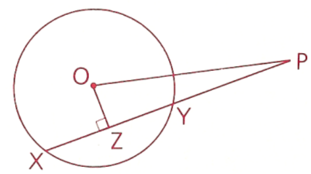
Find: The radii of the three ⓢ (Hint: This is a walk-around problem.)



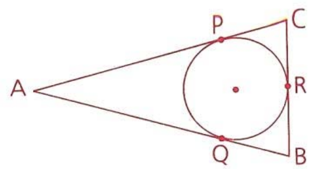
**17** The radius of $\odot$O is 10.
The secant segment $\overline{PX}$ measures 21 and is 8 units from the center of the $\odot$.

**a** Find the external part (PY) of the secant segment.

**b** Find OP.




**18** Given: $\triangle$ABC is isosceles, with base $\overline{BC}$.
Conclusion: $\overline{BR} \cong \overline{RC}$




**19** If two of the seven circles are chosen at random, what is the probability that the chosen pair are

**a** Internally tangent?

**b** Externally tangent?

**c** Not tangent?