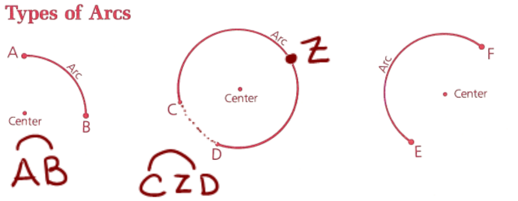
Name Stu Dent AMDG Ms. Kresovic
Adv Geo - 8 W 3/16/16 10-3: Arcs of a Circle

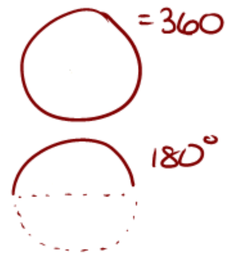
Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

Types of Arcs

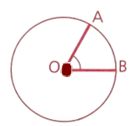




Definition An *arc* consists of two points on a circle and all points on the circle needed to connect the points by a single path.

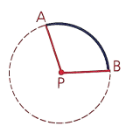
Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A *central angle* is an angle whose vertex is at the center of a circle.



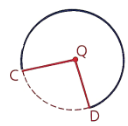
Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

Definition A *minor arc* is an arc whose points are on or between the sides of a central angle.



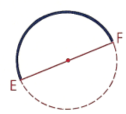
Central angle APB determines minor arc AB.

Definition A *major arc* is an arc whose points are on or outside of a central angle.



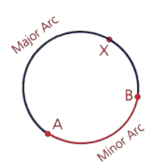
Central angle CQD determines major arc CD.

Definition A *semicircle* is an arc whose endpoints are the endpoints of a diameter.

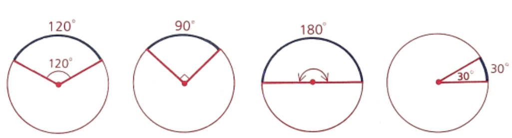


Arc EF is a semicircle.

The symbol $\frown$ is used to label arcs. The minor arc joining A and B is called $\overset{\frown}{AB}$. The major arc joining A and B is called $\overset{\frown}{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)



The Measure of an Arc = m central ∠

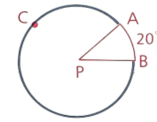


Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

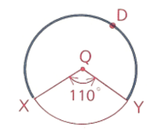
Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

Example

a Given: $m\overset{\frown}{AB} = 20$
Find: $m\overset{\frown}{ACB}$



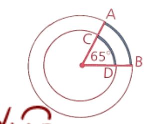
b Given: $m\angle XQY = 110$
Find: $m\overset{\frown}{XDY}$



$m\overset{\frown}{CD} = 65°$ & $m\overset{\frown}{AB} = 65°$

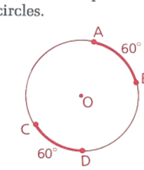
Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\overset{\frown}{AB} = 65$ and $m\overset{\frown}{CD} = 65$, but $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are not congruent. Under what conditions, do you think, will two arcs be congruent?

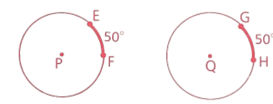


$\overset{\frown}{CD} \not\cong \overset{\frown}{AB}$

Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.



We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

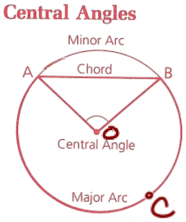


If ⊙P ≅ ⊙Q, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

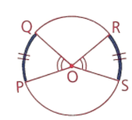
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.

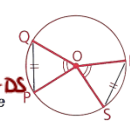


Theorem 79 *If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.* 2 ≅ central ∠s ⇒ 2 ≅ ARCS



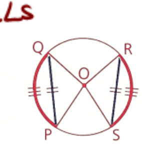
Theorem 80 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* 2 ≅ ARCS ⇒ 2 ≅ CENTRAL ∠s

Theorem 81 *If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* 2 ≅ CENTRAL ∠s ⇒ 2 ≅ CHDS



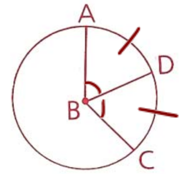
Theorem 82 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* 2 ≅ CHDS ⇒ 2 ≅ CENTRAL ∠S

Theorem 83 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* 2 ≅ ARCS ⇒ 2 ≅ CHDS



Theorem 84 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.* 2 ≅ CHDS ⇒ 2 ≅ ARCS

Problem 1 Given: $\odot$B;
D is the midpt. of $\widehat{AC}$.
Conclusion: $\overrightarrow{BD}$ bisects $\angle ABC$.

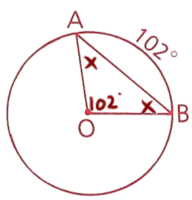


Proof

1 $\odot$B; D is the midpt. of $\widehat{AC}$.	1 Given
2 $\widehat{AD} \cong \widehat{DC}$	2 MDPT $\Rightarrow \cong$ ARCS (1)
3 $\angle ABD \cong \angle DBC$	3 2 $\cong$ ARCS $\Rightarrow$ 2 $\cong$ CENTRAL $\angle$s (2)
4 $\overrightarrow{BD}$ bisects $\angle ABC$.	4 2 $\cong \angle$s $\Rightarrow$ BISECTION (3)

Problem 2 If m$\widehat{AB}$ = 102 in $\odot$O, find m$\angle$A and m$\angle$B in $\triangle$AOB.

Solution $\widehat{AB} = 102°$



m$\angle$AOB = 102°	(m $\frown$ = m central $\angle$)
$\overline{OA} \cong \overline{OB}$	($\odot \Rightarrow \cong$ RADII)
$\angle A \cong \angle B$	($\cong$ $\triangle$ $\Rightarrow$ $\triangle$)
$102 + 2x = 180$	($\Sigma \angle$s $\triangle$ = 180°)
$2x = 78$	(Subtract)
$x = 39$	(Divide)

$\therefore$ m$\angle$A = m$\angle$B = 39°

Σ = Sigma means sum

Problem 3 **a** What fractional part of a circle is an arc of 36°? Of 200°?
b Find the measure of an arc that is $\frac{7}{12}$ of its circle.

Solution There are 360° in a whole $\odot$.

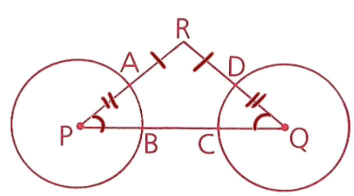
$\frac{\text{PART}}{\text{WHOLE}}$

a $\frac{36}{360} = \boxed{\frac{1}{10}}$; $\frac{200}{360} = \frac{20}{36} = \frac{10}{18} = \boxed{\frac{5}{9}}$ **b**

$\frac{7}{12} = \frac{x}{360} = \boxed{210°}$

30

Problem 4 Given: Ⓢ P and Q,
$\angle P \cong \angle Q$, $\overline{AR} \cong \overline{RD}$
Prove: $\widehat{AB} \cong \widehat{CD}$ (Hint: First prove that $\odot P \cong \odot Q$.)



Proof

1 Ⓢ P and Q	1 Given
2 $\angle P \cong \angle Q$	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 $\triangle \Rightarrow \cong$ (2)
4 $\overline{AR} \cong \overline{RD}$	4 Given
5 $\overline{AP} \cong \overline{DQ}$	5 Subtract (3, 4)
6 $\odot P \cong \odot Q$	6 $\cong$ RADII $\Rightarrow \cong$ Ⓢ (5)
7 $\widehat{AB} \cong \widehat{CD}$	7 $\cong$ CENTRAL $\angle$s OF $\cong$ Ⓢ $\Rightarrow \cong$ ARCS (2, 6)

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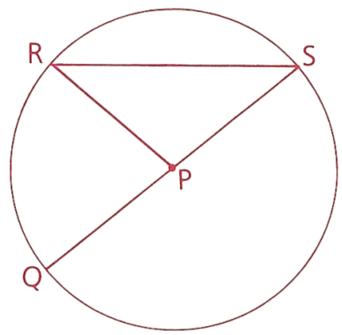
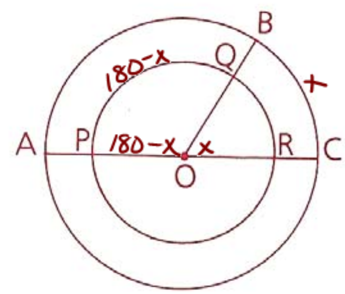
1 Match each item in the left column with the correct term in the right column.

a $\overset{\frown}{QRS}$ = 6
b $\overline{QS}$ = 2
c $\overset{\frown}{RQS}$ = 5
d $\overset{\frown}{RS}$ = 4
e $\overline{RS}$ = 3
f $\angle RPQ$ = 7
g $\overline{PS}$ = 1

1 Radius
2 Diameter
3 Chord
4 Minor arc
5 Major arc
6 Semicircle
7 Central angle



2 Given: Two concentric circles with center O; $\angle BOC$ is acute.



a Name a major arc of the smaller circle. = $\overset{\frown}{QPR}$

b Name a minor arc of the larger circle. $\overset{\frown}{BC}$

c What is $m\overset{\frown}{BC} + m\overset{\frown}{PQ}$? = $x + 180 - x = 180$

d Which is greater, $m\overset{\frown}{BC}$ or $m\overset{\frown}{PQ}$? $\overset{\frown}{PQ}$

e Is $\overset{\frown}{BC}$ congruent to $\overset{\frown}{QR}$?

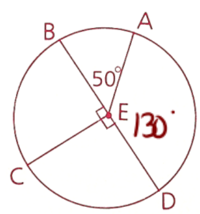
They have same measure but ⊙s ≇ ∴ No

m central ∠ = m ⌒

3 In circle E, find each of the following.

a $m\overset{\frown}{BC}$ = 90° c $m\overset{\frown}{ACD}$ 230° e $m\overset{\frown}{ADC}$ = 220°

b $m\overset{\frown}{AD}$ = 130° d $m\overset{\frown}{BAD}$ = 180°



e) $m\overset{\frown}{ADC} = \overset{\frown}{AD} + \overset{\frown}{DC} = 130° + 90°$

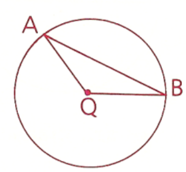
c) $\overset{\frown}{ACD} = \overset{\frown}{AB} + \overset{\frown}{BC} + \overset{\frown}{CD}$

$50 + 90 + 90 =$

180
50

4 Given: ⊙Q, $\angle A = 25°$

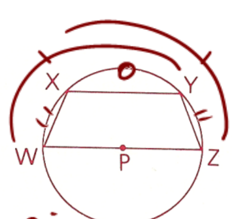
Find: $m\overset{\frown}{AB}$



5 Given: ⊙P,

$\overset{\frown}{WY} \cong \overset{\frown}{XZ}$

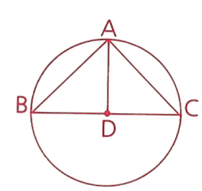
Conclusion: $\overline{WX} \cong \overline{YZ}$



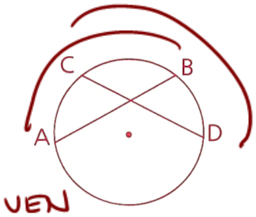
1. ⊙P, $\overset{\frown}{WY} \cong \overset{\frown}{XZ}$ 1. Given
2. $\overset{\frown}{XY} \cong \overset{\frown}{XY}$ 2. Ref
3. $\overset{\frown}{WX} \cong \overset{\frown}{ZY}$ 3. Subtract (1,2)
4. $\overline{WX} \cong \overline{YZ}$ 4. 2 ≅ ARCS ⇒ 2 ≅ CHDS

6 Given: ⊙D, $\angle B \cong \angle C$

Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{AC}$

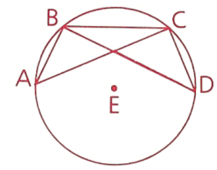


7 Given: $\overline{AB} \cong \overline{CD}$
Conclusion: $\widehat{AC} \cong \widehat{BD}$



1. $\overline{AB} \cong \overline{CD}$	1. GIVEN
2. $\widehat{AB} \cong \widehat{CD}$	2. 2 ≅ CHDS ⇒ 2 ≅ ARCS
3. $\widehat{CB} \cong \widehat{CB}$	3. Ref
4. $\widehat{AC} \cong \widehat{BD}$	4. Subtraction

8 Given: $\odot E$,
$\overline{AB} \cong \overline{CD}$
Prove: $\overline{BD} \cong \overline{AC}$



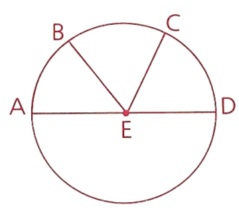
9 What fractional part of a circle is an arc that measures

a 8
b 240
c 144
d 315

10 Find the measure of an arc that is

a $\frac{3}{5}$ of its circle
b $\frac{5}{9}$ of its circle
c 70% of its circle

11 Given: $\overline{AD}$ is a diameter of $\odot E$.
C is the midpoint of $\widehat{BD}$.
$m\widehat{AB} = 9x + 30$,
$m\widehat{CD} = 54 - x$
Find: $m\angle AEC$



12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

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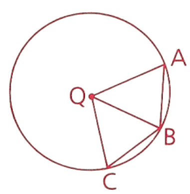
13 Find the length of each arc described. (The length is a fractional part of the circumference.)

a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

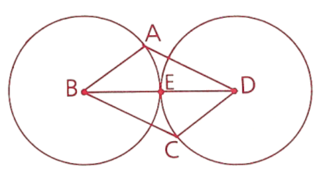
b An arc that has a measure of 270 and is part of a circle with radius 12

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\widehat{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.

15 Given: $\odot Q$;
B is the midpt. of $\widehat{AC}$.
Conclusion: $\angle A \cong \angle C$

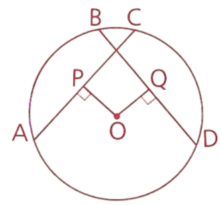


16 Given: $\odot B \cong \odot D$,
$\widehat{AE} \cong \widehat{CE}$
Prove: ABCD is a ▱.

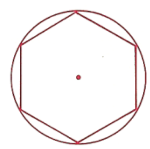


17 Given: $\odot O$,
$\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
$\overline{OP} \cong \overline{OQ}$

Conclusion: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$



18 A polygon is *inscribed in* a $\odot$ if all its vertices lie on the $\odot$. Find the measure of the arc cut off by a side of each of the following inscribed polygons.



a A regular hexagon

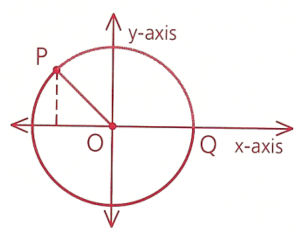
b A regular pentagon

c A regular octagon

19 Point P is located at $(-5, 5)$.

a Find the radius of $\odot O$.

b Find the measure of $\overset{\frown}{PQ}$.



20 Given: $\odot P \cong \odot Q$,
$\overline{BC} \cong \overline{CD}$

Conclusion: $\angle A \cong \angle E$