AMDG

Name STUDENT W 3/16/16 Ms. Kresovic

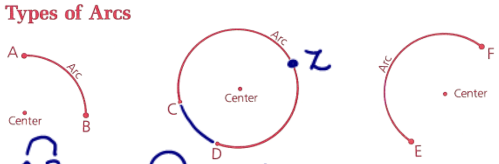
Adv Geo - 7 10-3: Arcs of a Circle

Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

Types of Arcs

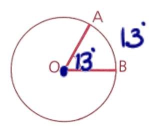


$\overset{\frown}{AB}$ 2 ltrs $\overset{\frown}{CZD}$ major ⇒ 3 ltrs

Definition An *arc* consists of two points on a circle and all points on the circle needed to connect the points by a single path.

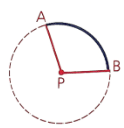
Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A *central angle* is an angle whose vertex is at the center of a circle.



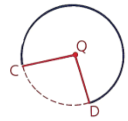
Radii $\overline{OA}$ and $\overline{OB}$ determine central angle AOB.

Definition A *minor arc* is an arc whose points are on or between the sides of a central angle.



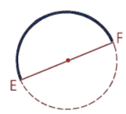
Central angle APB determines minor arc AB.

Definition A *major arc* is an arc whose points are on or outside of a central angle.



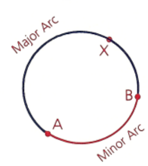
Central angle CQD determines major arc CD.

Definition A *semicircle* is an arc whose endpoints are the endpoints of a diameter.

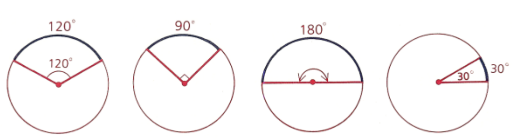


Arc EF is a semicircle.

The symbol ⌒ is used to label arcs. The minor arc joining A and B is called $\overset{\frown}{AB}$. The major arc joining A and B is called $\overset{\frown}{AXB}$. (The extra point, X, is named to make it clear that we are referring to the arc from A to B *by way of* point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)



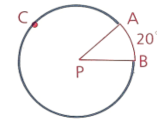
The Measure of an Arc



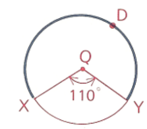
Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

Example **a** *Given:* $m\overset{\frown}{AB} = 20$ *Find:* $m\overset{\frown}{ACB}$

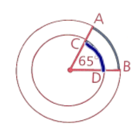


b *Given:* $m\angle XQY = 110$ *Find:* $m\overset{\frown}{XDY}$



* Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\overset{\frown}{AB} = 65$ and $m\overset{\frown}{CD} = 65$, but $\overset{\frown}{AB}$ and $\overset{\frown}{CD}$ are *not* congruent. Under what conditions, do you think, will two arcs be congruent?

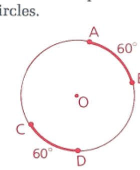


$m\overset{\frown}{CD} = 65°$

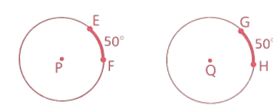
$m\overset{\frown}{AB} = 65°$

$\overset{\frown}{AB} \ncong \overset{\frown}{CD}$

Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.



We may conclude that $\overset{\frown}{AB} \cong \overset{\frown}{CD}$.

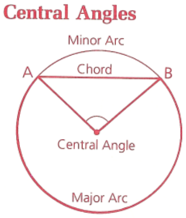


If ⊙P ≅ ⊙Q, we may conclude that $\overset{\frown}{EF} \cong \overset{\frown}{GH}$.

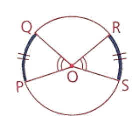
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.

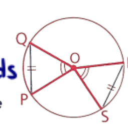


Theorem 79 *If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent.* 2 ≅ central ∠s ⇒ 2 ≅ arcs



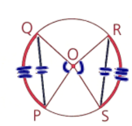
Theorem 80 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* 2 ≅ arcs ⇒ 2 ≅ central ∠s

Theorem 81 *If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* 2 ≅ central ∠s ⇒ 2 ≅ chds



Theorem 82 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent.* 2 ≅ chds ⇒ 2 ≅ central ∠s

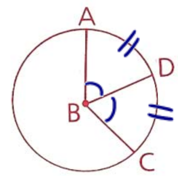
Theorem 83 *If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent.* 2 ≅ arcs ⇒ 2 ≅ chds



Theorem 84 *If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent.* 2 ≅ chds ⇒ 2 ≅ arcs

Problem 1

Given: $\odot$B;
D *is the midpt. of* $\overset{\frown}{AC}$.
Conclusion: $\overrightarrow{BD}$ *bisects* $\angle ABC$.

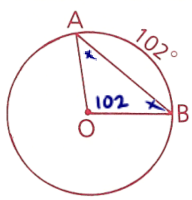


Proof

1 $\odot$B; D is the midpt. of $\overset{\frown}{AC}$.	1 Given
2 $\overset{\frown}{AD} \cong \overset{\frown}{DC}$	2 MDPT $\Rightarrow \cong$ ARCS (1)
3 $\angle ABD \cong \angle DBC$	3 2 $\cong$ ARCS $\Rightarrow$ 2 $\cong$ CENTRAL $\angle$S (2)
4 $\overrightarrow{BD}$ bisects $\angle ABC$.	4 2 $\cong$ $\angle$S $\Rightarrow$ BISECTION (3)

Problem 2 *If* $m\overset{\frown}{AB} = 102$ *in* $\odot$O, *find* $m\angle A$ *and* $m\angle B$ *in* $\triangle AOB$.

Solution $\overset{\frown}{AB} = 102°$



$m\overset{\frown}{AB} \Rightarrow m\angle AOB = 102°$ (m arc = m central $\angle$)
$\overline{OA} \cong \overline{OB}$ ($\odot \Rightarrow \cong$ radii)
$\angle A \cong \angle B$ ($\cong$ sides $\Rightarrow$ $\cong$ $\angle$s)
$102 + 2x = 180$ (Σ $\angle$s $\Delta = 180°$)
$2x = 78$ (subtract)
$x = 39$ (Divide)

Σ = Sigma means sum

$\therefore m\angle A = m\angle B = 39°$

Problem 3 **a** *What fractional part of a circle is an arc of 36°? Of 200°?*
b *Find the measure of an arc that is* $\frac{7}{12}$ *of its circle.*

$\frac{\text{PART}}{\text{WHOLE}}$

Solution There are 360° in a whole $\odot$.

a $\frac{36}{360} = \boxed{\frac{1}{10}}$ & $\frac{200}{360}$

$\frac{20}{36} = \frac{10}{18} = \boxed{\frac{5}{9}}$

b $\frac{7}{12} \cdot 360° =$

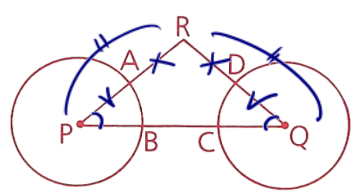
—OR—

$\frac{7}{12} = \frac{\boxed{210°}}{360}$

30

Problem 4

Given: Ⓢ P *and* Q,
$\angle P \cong \angle Q$, $\overline{AR} \cong \overline{RD}$
Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ (Hint: First prove that $\odot P \cong \odot Q$.)



Proof

1 Ⓢ P and Q	1 Given
2 $\angle P \cong \angle Q$	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 $\angle\angle \Rightarrow \triangle\triangle$ (2)
4 $\overline{AR} \cong \overline{RD}$	4 GIVEN
5 $\overline{AP} \cong \overline{DQ}$	5 Subtract (3 & 4)
6 $\odot P \cong \odot Q$	6 2 $\cong$ RADII $\Rightarrow$ 2 $\cong$ Ⓢ (5)
7 $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	7 2 $\cong$ central $\angle$s of 2 $\cong$ Ⓢ $\Rightarrow$ $\cong$ arcs (2,6)

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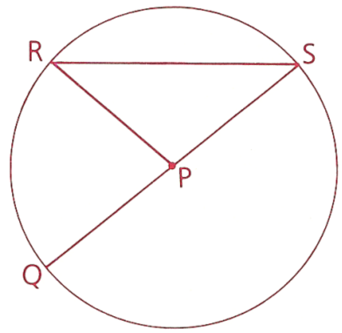
Adv Geo -

1 Match each item in the left column with the correct term in the right column.

a $\widehat{QRS}$ = 6		1 Radius
b $\overline{QS}$ 2		2 Diameter
c $\widehat{RQS}$ 5		3 Chord
d $\widehat{RS}$ 4		4 Minor arc
e $\overline{RS}$ 3		5 Major arc
f $\angle RPQ$ 7		6 Semicircle
g $\overline{PS}$ 1		7 Central angle



2 Given: Two concentric circles with center O; $\angle BOC$ is acute.

a Name a major arc of the smaller circle. $\widehat{QPR}$

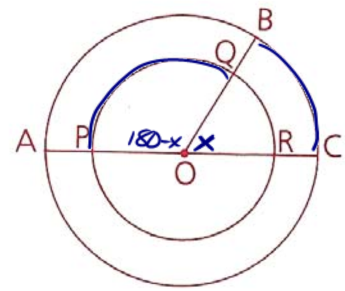
b Name a minor arc of the larger circle. $\widehat{BC}$

c What is $m\widehat{BC} + m\widehat{PQ}$? = $x + 180 - x = 180°$

d Which is greater, $m\widehat{BC}$ or $m\widehat{PQ}$? $m\widehat{PQ}$

e Is $\widehat{BC}$ congruent to $\widehat{QR}$?

No ∵ they are not in ≅ ⓢ even though they have the same measure.



m central ∠ = m arc

3 In circle E, find each of the following.

a $m\widehat{BC}$ = 90°

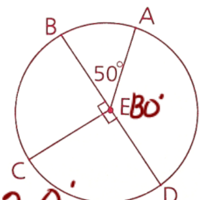
b $m\widehat{AD}$ = $180 - 50 = 130°$

c $m\widehat{ACD}$ $180 + 50 = 230°$

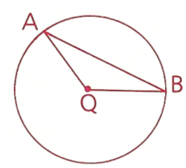
d $m\widehat{BAD}$ = 180°

e $m\widehat{ADC}$

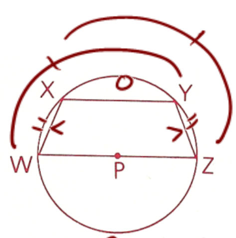
$\widehat{AD} + \widehat{DC} =$
$130° + 90 = 220$



4 Given: ⊙Q, $\angle A = 25°$
Find: $m\widehat{AB}$

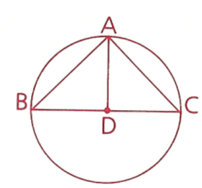


5 Given: ⊙P,
$\widehat{WY} \cong \widehat{XZ}$
Conclusion: $\overline{WX} \cong \overline{YZ}$

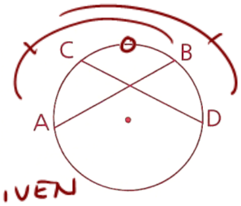


1. ⊙P, $\widehat{WY} \cong \widehat{XZ}$ — 1. GIVEN
2. $\widehat{XY} \cong \widehat{XY}$ — 2. Ref
3. $\widehat{WX} \cong \widehat{ZY}$ — 3. Subtract (1, 2)
4. $\overline{WX} \cong \overline{YZ}$ — 4. 2 ≅ ARCS ⇒ 2 ≅ CHDS (3)

6 Given: ⊙D, $\angle B \cong \angle C$
Conclusion: $\widehat{AB} \cong \widehat{AC}$

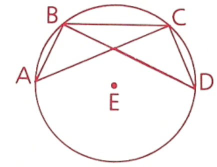


7 Given: $\overline{AB} \cong \overline{CD}$
Conclusion: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$



1. $\overline{AB} \cong \overline{CD}$ — 1. GIVEN
2. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$ — 2. 2 ≅ CHDS ⇒ 2 ≅ ARCS (1)
3. $\overset{\frown}{CB} \cong \overset{\frown}{CB}$ — 3. REF
4. $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ — 4. SUBTRACT

8 Given: $\odot E$,
$\overline{AB} \cong \overline{CD}$
Prove: $\overline{BD} \cong \overline{AC}$



9 What fractional part of a circle is an arc that measures

a 8 $\quad \frac{8}{360} = \frac{2}{90} = \boxed{\frac{1}{45}}$

~~**b** 240~~

c 144 $\quad \frac{144}{360} = \frac{\cancel{12}\cdot\cancel{12}}{\cancel{12}\cdot\cancel{30}} = \frac{6\cdot 2}{6\cdot 5} = \boxed{\frac{2}{5}}$

~~**d** 315~~

b) $\frac{240}{360} = \frac{12}{18} = \boxed{\frac{2}{3}}$

d) $\frac{315}{360} = \frac{7}{8}$

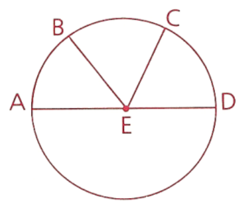
10 Find the measure of an arc that is

a $\frac{3}{5}$ of its circle

b $\frac{5}{9}$ of its circle

c 70% of its circle

11 Given: $\overline{AD}$ is a diameter of $\odot E$.
C is the midpoint of $\overset{\frown}{BD}$.
$m\overset{\frown}{AB} = 9x + 30$,
$m\overset{\frown}{CD} = 54 - x$
Find: $m\angle AEC$



12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

NAME

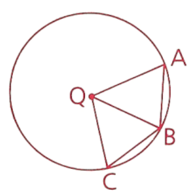
13 Find the length of each arc described. (The length is a fractional part of the circumference.)

a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12

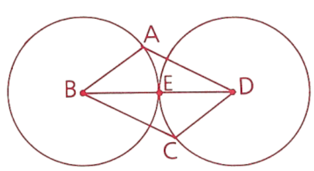
b An arc that has a measure of 270 and is part of a circle with radius 12

14 $\overline{AB}$ is a chord of circle E, and C is the midpoint of $\widehat{AB}$. Prove that $\overleftrightarrow{EC}$ is the perpendicular bisector of chord $\overline{AB}$.

15 Given: $\odot Q$;
B is the midpt. of $\widehat{AC}$.
Conclusion: $\angle A \cong \angle C$

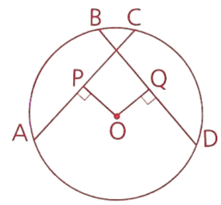


16 Given: $\odot B \cong \odot D$,
$\widehat{AE} \cong \widehat{CE}$
Prove: ABCD is a $\square$.



17 Given: $\odot O$,
$\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
$\overline{OP} \cong \overline{OQ}$

Conclusion: $\widehat{AB} \cong \widehat{CD}$

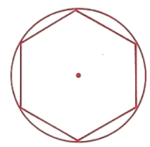


18 A polygon is *inscribed in* a $\odot$ if all its vertices lie on the $\odot$. Find the measure of the arc cut off by a side of each of the following inscribed polygons.

a A regular hexagon

b A regular pentagon

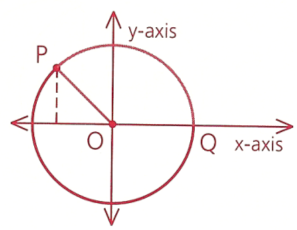
c A regular octagon



19 Point P is located at $(-5, 5)$.

a Find the radius of $\odot O$.

b Find the measure of $\widehat{PQ}$.



20 Given: $\odot P \cong \odot Q$,
$\overline{BC} \cong \overline{CD}$

Conclusion: $\angle A \cong \angle E$