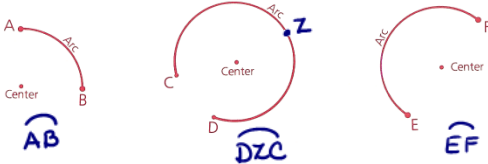


Objectives

After studying this section, you will be able to

- Identify the different types of arcs
- Determine the measure of an arc
- Recognize congruent arcs
- Apply the relationships between congruent arcs, chords, and central angles

Types of Arcs



Definition An arc consists of two points on a circle and all points on the circle needed to connect the points by a single path.

Definition The center of an arc is the center of the circle of which the arc is a part.

Definition A central angle is an angle whose vertex is at the center of a circle.

Radii \overline{OA} and \overline{OB} determine central angle AOB.

Definition A minor arc is an arc whose points are on or between the sides of a central angle.

Central angle APB determines minor arc \widehat{AB} .

Definition A major arc is an arc whose points are on or outside of a central angle.

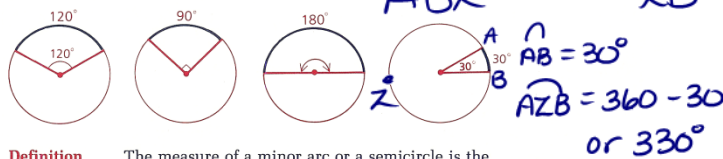
Central angle CQD determines major arc \widehat{CZD} .

Definition A semicircle is an arc whose endpoints are the endpoints of a diameter.

Arc EF is a semicircle.

The symbol $\widehat{}$ is used to label arcs. The minor arc joining A and B is called \widehat{AB} . The major arc joining A and B is called \widehat{AXB} . (The extra point, X, is named to make it clear that we are referring to the arc from A to B by way of point X. This helps to avoid confusion when a major arc or a semicircle is being discussed.)

The Measure of an Arc



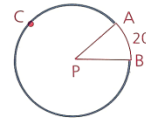
Definition The measure of a minor arc or a semicircle is the same as the measure of the central angle that intercepts the arc.

Definition The measure of a major arc is 360 minus the measure of the minor arc with the same endpoints.

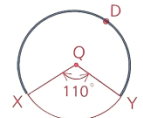
$\bigcirc = 360^\circ$

Example

a Given: $m\widehat{AB} = 20$
 Find: $m\widehat{ACB}$

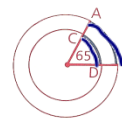


b Given: $m\angle XQY = 110$
 Find: $m\widehat{XDY}$



Congruent Arcs

Two arcs that have the same measure are not necessarily congruent arcs. In the concentric circles shown, $m\widehat{AB} = 65$ and $m\widehat{CD} = 65$, but \widehat{AB} and \widehat{CD} are not congruent. Under what conditions, do you think, will two arcs be congruent?



Definition Two arcs are congruent whenever they have the same measure and are parts of the same circle or congruent circles.

} same length



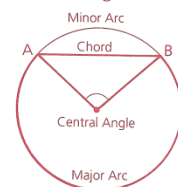
We may conclude that $\widehat{AB} \cong \widehat{CD}$.

If $\odot P \cong \odot Q$, we may conclude that $\widehat{EF} \cong \widehat{GH}$.

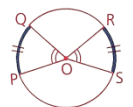
Relating Congruent Arcs, Chords, and Central Angles

In the diagram, points A and B determine one central angle, one chord, and two arcs (one major and one minor).

You can readily prove the following theorems.



Theorem 79 If two central angles of a circle (or of congruent circles) are congruent, then their intercepted arcs are congruent. $\cong \text{central } \angle s \Rightarrow \cong \text{arcs}$



Theorem 80 If two arcs of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent. $\cong \text{arcs} \Rightarrow \cong \text{central } \angle s$

Theorem 81 If two central angles of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent. $\cong \text{central } \angle s \Rightarrow \cong \text{chds}$



Theorem 82 If two chords of a circle (or of congruent circles) are congruent, then the corresponding central angles are congruent. $\cong \text{chds} \Rightarrow \cong \text{central } \angle s$

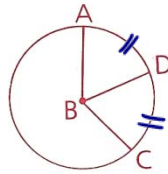
Theorem 83 If two arcs of a circle (or of congruent circles) are congruent, then the corresponding chords are congruent. $\cong \text{arcs} \Rightarrow \cong \text{chds}$



Theorem 84 If two chords of a circle (or of congruent circles) are congruent, then the corresponding arcs are congruent. $\cong \text{chds} \Rightarrow \cong \text{arcs}$

Problem 1

Given: $\odot B$;
 D is the midpt. of \widehat{AC} .
 Conclusion: \overrightarrow{BD} bisects $\angle ABC$.



Proof

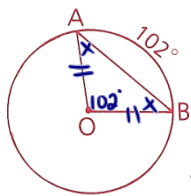
1 $\odot B$; D is the midpt. of \widehat{AC} .	1 Given
2 $\widehat{AD} \cong \widehat{DC}$	2 midpt \Rightarrow 2 \cong arcs (1)
3 $\angle ABD \cong \angle DBC$	3 2 \cong arcs \Rightarrow 2 \cong central \angle s (2)
4 \overrightarrow{BD} bisects $\angle ABC$.	4 2 \cong \angle s \Rightarrow bis (3)

Problem 2

If $m\widehat{AB} = 102$ in $\odot O$, find $m\angle A$ and $m\angle B$ in $\triangle AOB$.

Solution

$\widehat{AB} = 102^\circ$



$\widehat{AB} = 102 \Rightarrow m\angle AOB = 102^\circ$ ($m\cap = m\text{cent}\angle$)
 $\overline{AO} \cong \overline{OB}$ ($\odot \Rightarrow \cong$ radii)
 $\angle A \cong \angle B$ ($\triangle \Rightarrow \triangle$)
 $102 + 2x = 180$ ($\Sigma \angle$ s $\triangle = 180^\circ$)
 $2x = 78$ (Subtract)
 $x = 39^\circ$ (Divide) $\therefore m\angle A = m\angle B = 39^\circ$

$\Sigma =$ Sigma means sum

Problem 3

- a What fractional part of a circle is an arc of 36° ? Of 200° ?
- b Find the measure of an arc that is $\frac{7}{12}$ of its circle.

part
whole

Solution

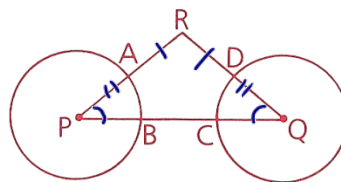
There are 360° in a whole \odot .

a $\frac{36}{360} = \frac{1}{10}, \frac{200}{360} = \frac{5}{9}$

b $\frac{7}{12} = \frac{x}{360} \Rightarrow 7 \frac{30}{12} = \frac{1}{2} x$
 $x = 210^\circ$
 or $\frac{7}{12} \cdot 360^\circ = 210^\circ$

Problem 4

Given: $\odot P$ and Q ,
 $\angle P \cong \angle Q$, $\overline{AR} \cong \overline{RD}$
 Prove: $\widehat{AB} \cong \widehat{CD}$ (Hint: First prove that $\odot P \cong \odot Q$.)



Proof

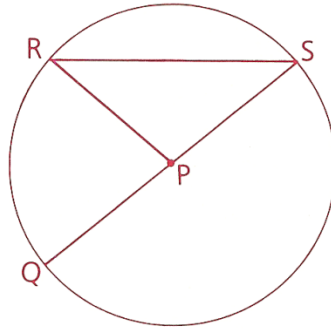
1 $\odot P$ and Q	1 Given
2 $\angle P \cong \angle Q$	2 Given
3 $\overline{RP} \cong \overline{RQ}$	3 $\triangle \Rightarrow \triangle$ (2)
4 $\overline{AR} \cong \overline{RD}$	4 Given
5 $\overline{AP} \cong \overline{DQ}$	5 Subtract (3 & 4)
6 $\odot P \cong \odot Q$	6 \cong radii $\Rightarrow \cong$ (5)
7 $\widehat{AB} \cong \widehat{CD}$	7 2 \cong cent \angle s of \cong (5) $\Rightarrow \cong$ arcs

Name
Adv Geo -

Ms. Kresovic

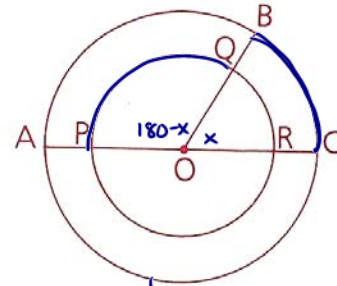
1 Match each item in the left column with the correct term in the right column.

- | | |
|-----------------------|-----------------|
| a $\widehat{QRS} = 6$ | 1 Radius |
| b \overline{QS} | 2 Diameter |
| c \widehat{RQS} | 3 Chord |
| d \widehat{RS} | 4 Minor arc |
| e \overline{RS} | 5 Major arc |
| f $\angle RPQ$ | 6 Semicircle |
| g \overline{PS} | 7 Central angle |

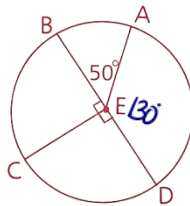


2 Given: Two concentric circles with center O;
 $\angle BOC$ is acute.

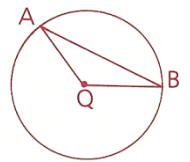
- a Name a major arc of the smaller circle. \widehat{QRP}
- b Name a minor arc of the larger circle. \widehat{BC}
- c What is $m\widehat{BC} + m\widehat{PQ}$? $= 180^\circ$
- d Which is greater, $m\widehat{BC}$ or $m\widehat{PQ}$? $m\widehat{PQ}$
- e Is \widehat{BC} congruent to \widehat{QR} ? same meas but not \cong \odot : No



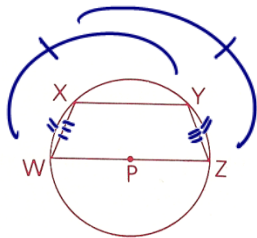
- 3 In circle E, find each of the following.
- a $m\widehat{BC} = 90^\circ$ c $m\widehat{ACD} = 230^\circ$ e $m\widehat{ADC} = 220^\circ$
- b $m\widehat{AD} = 130^\circ$ d $m\widehat{BAD} = 180^\circ$
- $m\widehat{ACD} = \widehat{DB} + \widehat{BA}$
 $180 + 50$
- $m\widehat{CD} + m\widehat{DA}$
 $90 + 130$



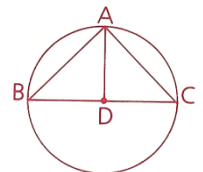
4 Given: $\odot Q$, $\angle A = 25^\circ$
Find: $m\widehat{AB}$



5 Given: $\odot P$,
 $\widehat{WY} \cong \widehat{XZ}$
Conclusion: $\overline{WX} \cong \overline{YZ}$

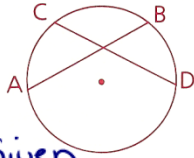


6 Given: $\odot D$, $\angle B \cong \angle C$
Conclusion: $\widehat{AB} \cong \widehat{AC}$



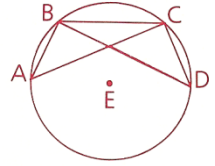
- | | |
|--|--|
| 1. $\odot P$, $\widehat{WY} \cong \widehat{XZ}$ | 1. Given |
| 2. $\widehat{WX} \cong \widehat{YZ}$ | 2. Subtract |
| 3. $\overline{WX} \cong \overline{YZ}$ | 3. \cong arcs \Rightarrow \cong chds |

7 Given: $\overline{AB} \cong \overline{CD}$
 Conclusion: $\widehat{AC} \cong \widehat{BD}$



1. $\overline{AB} \cong \overline{CD}$ 1. Given
2. $\widehat{AB} \cong \widehat{CD}$ 2. \cong chds \Rightarrow \cong arcs
3. $\widehat{AC} \cong \widehat{BD}$ 3. subtract

8 Given: $\odot E$,
 $\overline{AB} \cong \overline{CD}$
 Prove: $\overline{BD} \cong \overline{AC}$

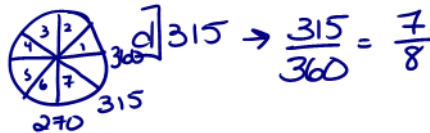


9 What fractional part of a circle is an arc that measures

a 8 $\frac{8}{360} = \frac{1}{45}$

c 144 $\frac{144}{360} = \frac{12}{30} = \frac{2}{5}$

b) 240 $\rightarrow \frac{240}{360} = \frac{2}{3}$



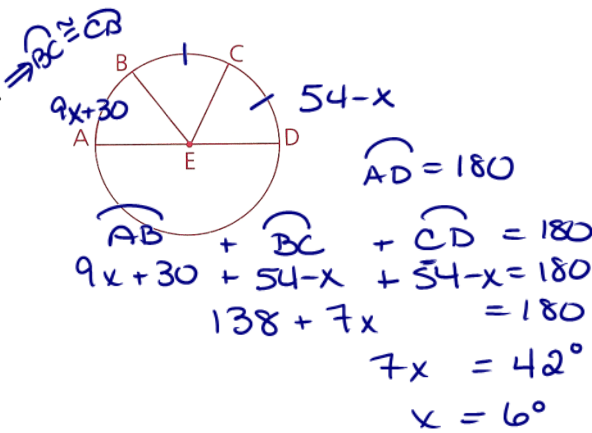
10 Find the measure of an arc that is

a $\frac{3}{5}$ of its circle
 $\frac{3}{5} = \frac{72}{360} \rightarrow 216^\circ$

b $\frac{5}{9}$ of its circle
 $\frac{5}{9} = \frac{40}{360} \rightarrow 200^\circ$

c 70% of its circle
 $\frac{70}{100} = \frac{7}{10} = \frac{210+42}{360} = 252^\circ$

11 Given: \overline{AD} is a diameter of $\odot E$.
 C is the midpoint of \widehat{BD} .
 $m\widehat{AB} = 9x + 30$,
 $m\widehat{CD} = 54 - x$



Find: $m\angle AEC =$
 $\widehat{AB} + \widehat{BC}$
 $9x + 30 + 54 - x$
 $9(6) + 30 + 54 - 6$

$\widehat{AD} = 180$
 $\widehat{AB} + \widehat{BC} + \widehat{CD} = 180$
 $9x + 30 + 54 - x + 54 - x = 180$
 $138 + 7x = 180$
 $7x = 42^\circ$
 $x = 6^\circ$

12 Find the length of a chord that cuts off an arc measuring 60 in a circle with a radius of 12.

NAME

13 Find the length of each arc described. (The length is a fractional part of the circumference.)

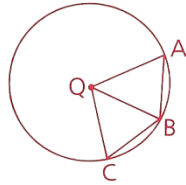
- a An arc that is $\frac{5}{8}$ of the circumference of a circle with radius 12
- b An arc that has a measure of 270 and is part of a circle with radius 12

14 \overline{AB} is a chord of circle E, and C is the midpoint of \widehat{AB} . Prove that \overleftrightarrow{EC} is the perpendicular bisector of chord \overline{AB} .

15 Given: $\odot Q$;

B is the midpt. of \widehat{AC} .

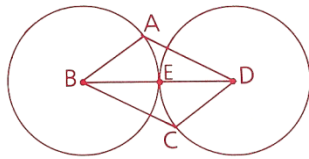
Conclusion: $\angle A \cong \angle C$



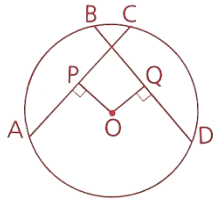
16 Given: $\odot B \cong \odot D$,

$\widehat{AE} \cong \widehat{CE}$

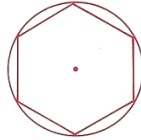
Prove: ABCD is a \square .



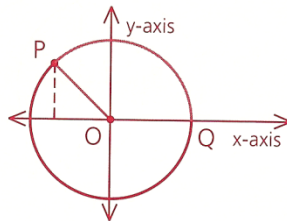
- 17 Given: $\odot O$,
 $\overline{OP} \perp \overline{AC}$, $\overline{OQ} \perp \overline{BD}$,
 $\overline{OP} \cong \overline{OQ}$
 Conclusion: $\widehat{AB} \cong \widehat{CD}$



- 18 A polygon is *inscribed* in a \odot if all its vertices lie on the \odot . Find the measure of the arc cut off by a side of each of the following inscribed polygons.
- a A regular hexagon
 - b A regular pentagon
 - c A regular octagon



- 19 Point P is located at $(-5, 5)$.
- a Find the radius of $\odot O$.
 - b Find the measure of \widehat{PQ} .



- 20 Given: $\odot P \cong \odot Q$,
 $\overline{BC} \cong \overline{CD}$
 Conclusion: $\angle A \cong \angle E$

